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LOGIC FOR CHILDREN,

DEDUCTIVE AND INDUCTIVE.

BEING THE SUBSTANCE OF

TWO ADDRESSES TO TEACHERS

DELIVERED BEFORE THE COLLEGE OF PRECEPTORS.

ENTITLED

1. ON A METHOD OF TEACHING DEDUCTIVE OR FORMAL LOGIC TO CHILDREN BY MEANS OF WORDS AND COUNTERS, 8 MAY, 1872.
2. ON SCHOOL INDUCTIONS, 12 FEB., 1873.

BY

ALEXANDER J. ELLIS,

F.R.S., F.S.A., F.C.P.S., F.C.P.,

PRESIDENT (FOR THE SECOND TIME) OF THE PHILOLOGICAL SOCIETY, MEMBER OF THE MATHEMATICAL SOCIETY, FORMERLY A VICE-PRESIDENT OF THE COLLEGE OF PRECEPTORS, AND FORMERLY SCHOLAR OF TRINITY COLLEGE, CAMBRIDGE, B.A. 1837.

The first lecture reprinted from the EDUCATIONAL TIMES *for June, July, and August*, 1872; *the second from the same for March, April, and May*, 1873; *both with numerous additions.*

PRINTED OFF IN 1872 AND 1873, AND PUBLISHED IN 1882.

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NOTICE.

THESE pages are for Teachers only, or for Adult Students. They should on no account be put into the hands of children.

Their object is to shew Teachers who have studied other works (as those cited on p. 46, note 5, to which I would now add: *W. Stanley Jevons*, Studies in Deductive Logic, a Manual for Students, Macmillan, 1880, pp. 304) how to teach Deductive Logic with the greatest amount of simplicity, combined with scientific accuracy, and to instil the principles of Inductive Logic into the minds of the youngest children. The notes are for the further guidance of teachers or others who wish to pursue the study for themselves.

The Deductive Logic was printed off in 1872, and has remained untouched ever since. In 1873 an Appendix on some points of Higher Deductive Logic, applying my methods to everything in De Morgan's and Boole's works, was written, and the delay in publication arose from an intention to revise this Appendix. But so much has since appeared on the subject which ought to be included, that the idea of printing such an Appendix has been definitively abandoned, and the tract is published as it stood in 1872. It is believed that the exemplifications by the vowels in words, notation, diagrams, and working of the syllogism, here explained, are original, while they appear to be the simplest and most complete hitherto proposed. In the abandoned Appendix it was shewn that the same methods were applicable for the solution of the highest logical problems.

The Inductive Logic, printed in 1873, was published separately in that year, with some slight alterations, under the title "On School Inductions, or how to familiarise school children with the principles of Inductive Logic." It now appears as it was originally written.

ALEX. J. ELLIS.

25, ARGYLL ROAD,
KENSINGTON, LONDON, W.
April, 1882.

ERRATA.

p. 12, line last, diagram (2), *for* ƎV in the left-hand square, *read* Ǝ*v*.
p. 53, line 1, *for* $\ddagger_1 (x.y,\ \ x.y)$ *read* $\ddagger_1 (x.Y,\ \ x.y)$.
p. 91, foot-note, last lines of paragraphs 10, 11, and 12, *for* p. 26 *read* p. 86; and last line of paragraph 14, *for* p. 24 *read* p. 84.

To prevent trouble, the Reader is requested to correct these trifling, but very confusing, errors before reading the pages in question.

A. J. E.

CONTENTS.

Part I.—DEDUCTIVE LOGIC.

Part II.—INDUCTIVE LOGIC.

LOGIC FOR CHILDREN.

1. Goethe, in that biting satire upon University studies, to which he gives the shape of advice from Mephistopheles to a Student, makes his fiend say:—"My dear friend, I advise you, first of all, to attend a course of logic. There your mind will be properly drilled, and laced up in the tightest of boots, so that it may henceforth slink more circumspectly along the path of thought, and not perchance will-o'-the-wisp-it hither and thither, through thick and thin. Then you'll be taught for many a day, that what you've been in the habit of doing in a moment, as easy as eating and drinking, must have its One! Two! Three! To be sure, the manufacture of thought is like a masterpiece of weaving, where one treadle moves a thousand threads. The shuttles fly hither and thither, the threads flow unseen, a single stroke strikes a thousand combinations. Your philosopher comes up, and proves to you that it can't help being so. 'The first was so, the second so, and hence the third and fourth were so; and if the first and second had not been there, the third and fourth would have never come to pass.' The students go into ecstacies over his explanation,—but none of them have become weavers."

2. Now this is very wittily and pungently expressed, and the truth at the bottom of it all is incontestable. Thinking by rule will not make an original thinker. But then, however original a thinker may be, he can only think according to rule, for the rule is merely the register of the processes actually followed by original thinkers. We don't expect, and we don't desire, every human being born into the world to have the insight of a Goethe,—not to mention an Aristotle, who, himself the towering mind of his day, first bethought himself of the advantage that would accrue from accurately stating the processes and laws of reasoning. But every one of us, from the grandson to the grandsire, has to go through small processes of reasoning every hour of his life. Every one of us, every time he speaks, has to make an assertion. He seldom does speak without making several assertions, which in some way modify and limit one another. And in point of fact, many of us are often apt to make assertions without knowing precisely what

each of them means separately, and still more often without knowing how they limit previous assertions, or even how to find out the extent of limitation. Hence we are all prone to make blunders, sometimes simply ridiculous, often mortifying, and not unfrequently entailing other disagreeable consequences. Now if there are a few simple and general considerations which will help us out of this difficulty, should we grudge the labour of mastering them? To say that when reasoners have thought out processes of reasoning, they should not be studied by those who wish to reason, seems about as wise as to say that because people can make themselves understood by picking up so much of a language as strikes their ear, they should not make use of a grammar and a dictionary, or that because all the rules of composition will never make a poet, no simple-minded soul who wishes to convey his thoughts intelligibly, and flounders awfully in the attempt, should avail himself of rules of composition.

3. Now, the College of Preceptors seems to have plainly stated its own opinion, by making a certain (albeit very small) amount of logical knowledge indispensable for passing their diploma examinations; and though their main thought was to guide the teacher himself in the application of methods of teaching, yet there must necessarily have been a secondary object in view referring to the matters taught. Clearly, of all others, children should be led to comprehend the expression of thought, and hence teachers should be put into a condition to train them. Of course, no one supposes that the barbarous terminology and unwieldy processes of scholastic logic, which seem very like sledge-hammers for killing flies, should be presented to the mind of a child, already so much overburdened with the pedantry of grammar. But are these necessary? Do we require more distinctions than objects distinguished? more classes than things classified? a dozen sesquipedalian words to express a single simple notion? difficult and subtle discriminations, in which the difficulty and subtlety mainly arise from viewing but half the battle? This is what shocks one in the old logic, which schoolmen have petted into a monster. 'Tis well that in recent times, so recent that they are within the experience of many here present, two great thinkers, whose loss we have had recently to deplore, Dr. Boole and Professor De Morgan, have shown us a way out of this muddle. It is from these two writers principally that I have drawn the materials for my own view of the real scope of so-called deductive logic. On working out a system in detail,* it struck me that the method was eminently adapted for school instruction, and that it could really be adapted, in its early stages, as a method of teaching logic to children. Of course it is impossible, during early school work, to do more than scrape the surface of so great a subject; but the soil is naturally so fertile, that I do not despair of a crop with even such shallow cultivation.

* In my paper, entitled "Contributions to Formal Logic," read before the Royal Society on Thursday, 25th April, 1872. See "Proceedings of the Royal Society," vol. 20, No. 134, p. 307.

4. Let me first confide to you, as a secret which you must strictly keep from your pupils, that "the intention of deductive logic is to determine the precise meaning of any number of given assertions—that is, to determine precisely what they affirm, precisely what they deny, and precisely what they leave in doubt, separately and jointly." This is the whole problem of deductive logic. It has nothing to do with the formation of assertions. That is the special object of induction. It has nothing directly to do with the correctness or incorrectness of the assertions. It takes them as they come, good, bad, and indifferent, and determines precisely what ground they cover, singly and in combination. Hence it must not be imputed as a fault, that logic leads to no new discovery; that the conclusion is involved in the premisses; that it merely tells us in other words, as a final assertion, what has been already sufficiently told in the preliminary assertions. This is no fault in logic; this is its aim. It is just because people do not in general know the full force of each assertion they launch, the number of cases which it settles, and more especially the number of cases which it leaves entirely unnoticed; it is just because people do not see the combined effect of their assertions, do not know what the premisses do and do not contain or imply; it is just for these reasons that it becomes of the last importance to have an easy, straightforward, infallible, and general method of finding all this out. Herein lies the *raison d'être* of deductive logic.

5. The value of this method, in indirectly determining the correctness of assertions, cannot be over-estimated. It is only when we see every case which an assertion does and does not cover, that we can confront it with fact. Hence, in every stage of induction, deduction is imperative. All induction consists in the formation of successive hypotheses, which have to be stated as assertions, and these hypotheses have to be put to the test of observation and experiment in the well-known methods. By deduction, and by deduction alone, can we be sure of conducting this test with the requisite thoroughness. Of course such applications imply a very much deeper knowledge of the subject than can be presented to children; but, if I mistake not, sufficient foundation can be laid in very youthful days for a superstructure which will prove advantageous to the most adventurous investigator of nature. To-night I shall only deal with two kinds of assertion, and I shall not advance far in their consideration; but the method I shall explain is general, and it will be found that most other problems of deduction differ from those now adduced in complication only, not in principle.

6. Language has become possible solely because "different attributes frequently reside at the same time in the same thing, and the same attribute generally resides at the same time in different things," where the word "things" is used with the utmost generality, for all objects of thought, such as material objects, their qualities and actions, states, conceptions, feelings, relations, in short, whatever can be named. Now, of course, no teacher would be mad enough to present children with such excessively abstract general propositions. But if he wishes to make his pupils think,

he must contrive some method of making them perfectly familiar with this fact, which pervades all language, and is the foundation of the first class of assertions that they will have to analyze, a class which occupies by far the greater part of all treatises on logic. Of course this must be effected by presenting the pupils with concrete objects of thought; and the method which occurs to me as most convenient to the teacher, and at the same time as the best introduction to the future expression of abstract logical relations, is the following. It is based on the supposition that a teacher would not be called upon to give lessons on logic to pupils who cannot read and write.

7. The teacher requests each pupil in his class to write down about twenty English words, each on a separate piece of paper. The choice of words should be left to the pupils, but for convenience of writing they should be recommended to choose "short" words. This done, let each pupil sort his words, putting on the left hand all those containing A, and all the rest on the right. Let this right-hand heap be well looked through to ascertain that not one of them contains A. All the words are therefore divided into two heaps, which we will call A-words, and non-A-words, and in writing we will employ a little *a* in place of the words "non-A", so that the second heap contains *a*-words only. If each pupil has been able to make this division, it will be felt that it can always be made. But perhaps one or two pupils will have no A-words, or no *a*-words. If this should not happen to be the case, make instances by requesting one pupil to throw away his A-words, and another pupil his *a*-words. Then an opportunity will be offered for showing that this was not a peculiarity of our language, because other pupils have found words of the missing kinds, but that it merely depended upon the choice of words, that is, upon the words thought of, or upon the "range of thought," a most useful and important conception. Then explain that, to get the most common, or the most general case, we ought to lay down as a rule or general condition, that when we have to look after A-words, we should have "at least one" A-word, and at least one *a*-word word, within the "range of thought," because the exceptional cases are very easily hunted up when the general cases are known (art. 20). Perhaps some word, as "papa, avail, abase," may contain more than one A. If no such words occur, let the teacher supply some on the black board, and ask to which set they belong. Clearly to the A-words, because they contain "at least one" A, and that is all we mean by an A-word. Hence, if we suppose the "general condition" just stated to be fulfilled,—*and whatever letters we talk about we shall hereafter suppose it to be fulfilled*—all the words within the range of thought can be separated into two sets, the first or A-words, containing "at least one" A, and the second or *a*-words, containing "not even one" A. No word can belong to both sets, which are perfectly exclusive one of the other. It is worth while bestowing great pains on this elementary conception, which is the basis of all classification, and which, expressed in the usual abstract language of logic, is intensely perplexing to a child's mind.

8. We now advance a stage. Take each of the two sets of A-words and *a*-words, and examine them for E. Observe that vowels are selected by the teacher because all words have some vowel or other. The A-words can then be divided into two heaps, containing at least one E, and containing not even one E. These will then be E-words and *e*-words. But in order to show that they have been formed out of the single set of A-words, prefix A to each, and call the sets AE-words, (read "A, E, words") and A*e*-words (read "A, non-E, words"). Do the same with the *a*-words, and thus find two other sets of *a*E-words, (read "non-A, E words,") and *ae*-words (read "non-A, non-E words"). Of course the general condition has been complied with, so that there is at least one E-word and one *e*-word. But it will not always happen, even then, that each pupil will have all four sets of AE, A*e*, *a*E, *ae*, words. Having ascertained whether any one has got all four sets, begin with him.

9. Then the AE-words and *a*E-words are two sets which could have been formed out of the one set of E-words, supposing that the list of words had been examined for E first. And the A*e* and *ae* are two sets which could have been formed out of the one set of *e*-words. This must not merely be acknowledged, but practically done. Let one pupil that has all the four sets of words read out his list, and let the teacher write it on the black board thus,—

	AE	A*e*	*a*E	*ae*
A	———	———	....	....
E	———	....	———	....
	ale	ail	feet	got
	bate	papa	sieve	put
	seat	cat	seize	mutton
	have	sham	conceive	slit
	amen	lady	people	round

The continuous lines group together all the A- and all the E-words; the dotted lines group together all the *a*- and all the *e*-words. This being understood, merely writing such lines with A and E at the beginning, will suffice to show, by overlapping, how the two different subdivisions can be made. Thus a mode of forming logical diagrams is suggested, which will be found of the greatest use.

10. Now observe that, instead of selecting words, it would have been sufficient to have written upon *at least one* of the slips of paper the letter A, implying that at least that one slip contained or represented a word containing A, and to have written on all the rest the letter *a*, implying that all these represented words without A, and then to have written E upon at least one of all these slips, and *e* upon all the rest, implying respectively that the slips represented words containing or not containing E. In this way every one of the slips would bear one or other of the marks AE, A*e*, *a*E, *ae*, and these marks would represent the "names" of these

words considered in relation to their containing or not containing the letters A and E. The names A and *a* would be "simple" names; but AE, A*e*, *a*E, *ae*, would be "compound" names, and may be briefly termed *compounds*. This being well understood and tried with various slips of paper,* will form the basis for a mechanical arrangement of great value in teaching. The *number* of slips of paper containing each "compound" is evidently unimportant for ascertaining the fact that there *is at least one* such compound. Hence four slips of paper—or, for greater convenience of handling, four slips of pasteboard, or "counters"—may be used, marked in bold letters with the names of the compounds AE, A*e*, *a*E, *ae*, respectively. For class teaching these counters should be six inches long and three high, and the capital letters should be more than two inches high, and very clear and bold. These are readily drawn with a camel-hair pencil dipped in *black* ink. The back of each counter, for a purpose immediately to be explained, should be marked with the same letters in *red* ink. If four other counters be marked with A, *a*, E, *e*, respectively, and little ledges be arranged for resting them on, the double arrangement may be shown thus,—

A		*a*		E		*e*	
AE	A*e*	*a*E	*ae*	AE	*a*E	A*e*	*ae*

So that, in fact, we have only to transpose A*e*, *a*E, in order to convert the first into the second. Be careful that the pupil understands that each of these counters stands for a *set* of words consisting of *at least one* word. Each counter therefore represents a column, with at least one word in it.§

11. Now suppose, as is most likely, that some of the pupils have not supplied words which can be fitted into every column, so that at least one column is empty. The question then arises (and it is one of primary importance), in how many different ways is it possible to have empty columns, consistent with the fulfilment of the "general condition"? This the pupils must be led to answer for themselves. Take a pupil that has only three sets. Suppose AE to be "absent," that is, that there is *not even one word* in the AE

* Begin by writing the compound names on the front or back of the actual slips of paper containing words; as, mate AE, met *a*E, &c.

§ A complete set of counters for the first kind of assertions consists of twenty-six pieces, namely A, *a*, E, *e*, I, *i*; AE, A*e*, *a*E, *ae*; AI, A*i*, *a*I, *ai*; EI, E*i*, *e*I, *ei*; AEI, AE*i*, A*e*I, A*ei*, *a*EI, *a*E*i*, *ae*I, *aei*; all with black letters on one side, and red on the other. Also a set of eleven *indices*, three inches high and one and a-half inch wide, black on one side only, namely 2_1, 2_1, 2_2, 2_2, 2_3, 2_3, 2_4, 2_4, 3_1, 3_2, 3_3. For the second set of assertions thirty pieces are required, namely, twenty-six black on one side and red on the other, X, *x*, Y, *y*, Z, *z*; X.Y, X.*y*, *x*.Y, *x*.*y*; X.Z, X.*z*, *x*.Z, *x*.*z*; Y.Z, Y.*z*, *y*.Z, *y*.*z*; X.Y.Z, X.Y.*z*, X.*y*.Z, X.*y*.*z*, *x*.Y.Z, *x*.Y.*z*, *x*.*y*.Z, *x*.*y*.*z*; and four black on one side only, X′, X″, *x*′, *x*″. The uses of these forms will be explained in due course. For classes of two or three persons, smaller counters, made on blank address cards, will suffice; and as these may be laid on a table, no ledges to support them are necessary. *Unless the reader will construct counters of some kind, and use them as directed, he cannot properly appreciate the method.*

column. Show this by turning the *red* side of the AE counter, which will therefore mean an *empty column*. Now is it quite necessary that when AE is absent, A*e*, *a*E, *ae*, should be all "present," that is, that there should be at least one word in each of these columns? Try this. What does the general condition demand? That there should be at least one A-word and one *a*-word. But what is implied by there being at least one A-word? All A-words, as we have seen, must be AE-words or A*e*-words. There are no AE-words, there must be, therefore, at least one A*e*-word. Hence A*e* must be present. The A*e* counter is therefore ranged with the *black* side out, beside the *red* AE. Again, there must be at least one E-word. But all E-words are either AE-words or *a*E-words. There are no AE-words. There must, therefore, be at least one *a*E-word. Place a black *a*E on the ledge. Then observe that, as we have A*e* and *a*E, we have already satisfied the general condition for *e*-words and *a*-words, and hence cannot tell whether there are any *ae*-words or not. There may be, or may not be, any; there must be at least one, or else not even one; but when we say there are no AE-words, we say nothing about the *ae*-words. Their presence is therefore "doubtful." Mark this by putting the *ae* counter on the ledge, black side out, but inverted, thus $\boxed{\text{ǝɐ}}$. This doubtful compound must be well understood. Revert to the general condition. There must be at least one *a*-word. All *a*-words are *a*E-words, or *ae*-words. We know that there is at least one *a*E-word, and this gives at least one *a*-word, because we only want a word without A, and whether it contains E or not is quite indifferent. Hence we cannot tell whether there are any *ae*-words from this consideration. Similarly from knowing that there must be at least one *e*-word, and that there is at least one A*e*-word, we cannot tell whether there is any *ae*-word or not. This is very important for all future applications of reasoning.

We see then that absent AE gives present A*e*, and present *a*E, but doubtful *ae*; but that in any real case that can be formed, the doubt will be resolved when all the words are known, and that there will be either present *ae* or absent *ae*. Cases, however, may well be imagined where all the words are *not* known, but only the fact that none contain both A and E, and these, in the applications, are the usual cases.

12. Now the counters are very convenient, nay, almost indispensable, in teaching, but for registering the results we require other signs. We cannot use the distinction of upright black letters for *present*, inverted black for *doubtful*, and red for *absent* compounds. Hence it is convenient to use abbreviations for these words. In my Complete System of Logical Notation, I have found it most convenient to use—

‡ for *present*, ? for *doubtful*, and † for *absent*,—

as shown in the following cases, where the exact meaning of each symbol should be rendered quite plain, and no use of "some" for "at least one," or "no" for "not even one," should be allowed, as the words "some, no" have occasioned great confusion.

‡AE, read "present, A,E," means that "there is at least one word within the range of thought which contains at least one A and at least one E."

‡A*e*, read "present, A, non-E," means that "there is at least one word within the range of thought which contains at least one A and not even one E."

‡*a*E, read "present, non-A, E," means that "there is at least one word within the range of thought which contains at least one E and not even one A."

‡*ae*, read "present, non-A, non-E," means that "there is at least one word within the range of thought which contains not even one A, and also not even one E."

†AE, †A*e*, †*a*E, †*ae*, read respectively "absent, A, E," "absent, A, non-E," "absent, non-A, E," and "absent, non-A, non-E," mean that "there is not even one word within the range of thought which contains, respectively, at least one A and at least one E; at least one A and not even one E; at least one E and not even one A; and not even one A and not even one E."

While ?AE, ?A*e*, ?*a*E, ?*ae*, would imply a doubt arising from the absence of information as to whether ‡AE or †AE, whether ‡A*e* or †A*e*, whether ‡*a*E or †*a*E, whether ‡*ae* or †*ae* respectively.

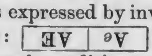
To embrace the cases where the presence or absence of every single one of the four compounds is declared, it is convenient to compound these signs, thus—

‡‡ for *all present with*, or *pre-present*,
‡† for *all present but*, or *pre-absent*,
†† for *one absent with*, or *ab-absent*,

the precise use of which is shown in the next article.

We have already seen that the general condition that there should be at least one A-word, obliges us to acknowledge that there must be at least one AE-word, or else at least one A*e*-word, and that there may be more than one of either or both sets. That is, at least one of the compounds AE, A*e* must be present, and both may be present. That is, we may have ‡AE and ‡A*e*, or ‡AE and †A*e*, or †AE and ‡A*e*, but cannot have †AE and †A*e*. This very curious limitation is constantly occurring in logic, and requires a sign. It is expressed by—

‡¹ (AE, A*e*) read "present at least one of AE and A*e*."

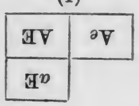
With the counters it is expressed by inverting both, and placing them close together thus: .

Observe that the general conditions give *four* such combinations, called *limitates*, namely,

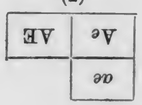
‡¹(AE, A*e*), ‡¹(AE, *a*E), ‡¹(A*e*, *ae*), ‡¹(*a*E, *ae*.)

Combinations of these may be indicated by groupings of the counters. Thus the two first might be (1), and the two last might be (2), while the whole four sets might be (3).

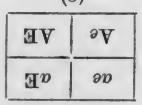
(1)



(2)



(3)



The inversion shows the really doubtful nature of each individual compound, and the grouping shows the limitation of that doubt. Another method of marking this fact is given in art. 26.

The following signs are also useful in writing:

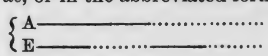
|| for *means*, and
(·) for *and, with, together with.*

These signs must only be introduced gradually as a means of easily writing what the counters indicate, and the pupils must be exercised in passing from the written forms to the counters, and thence to the sets of words themselves, as the real things under consideration.

13. We may now note all the cases which can possibly happen regarding the separation of words into four sets, and each case must be rigorously demonstrated by the pupils themselves, in the manner described in art. 11. The continuous and dotted lines show immediately how the columns would overlap, and what columns would be absent; but each case should also be illustrated by words, and, if possible, the actual cases which occur among the pupils should be taken first; if these are not enough, the pupils, rather than the teacher, should be led to furnish the rest. Pupils should also be exercised in constructing diagrams for themselves, giving various lengths to the dotted and continuous lines, interrupting dotted by continuous lines, and so on, till they feel that the length of these lines is of no consequence, (because they merely indicate the *breadth* of columns containing words; and as the *length* of those columns is indefinite, any number of words can be written in any column, however narrow,) but that the overlapping of the dotted and continuous parts in the two lines is everything, (because each overlapping shows the presence of a certain set of words).

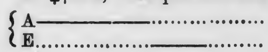
The Seven Possible Arrangements (or Complete Assertions).

i. ‡‡AE, "all present," also written ‡‡A*e*, ‡‡*a*E, or ‡‡*ae*, *means* present AE, and present A*e*, and present *a*E, and present *ae*, or in the abbreviated form, || ‡AE · ‡A*e* · ‡*a*E · ‡*ae* ;.



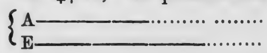
that is, at least one word occurs in each one of all the four sets.

ii. ‡†AE, "all present but AE," || †AE · ‡A*e* · ‡*a*E · ‡*ae*;



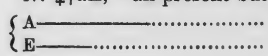
there is not even one word of the first set, but there is at least one word in each of the other three sets.

iii. ‡†A*e*, "all present but A*e*," || ‡AE · †A*e* · ‡*a*E · ‡*ae*;



there is not even one word of the second set, but there is at least one word in each of the other three sets.

iv. ‡†*a*E, "all present but *a*E," || ‡AE · ‡A*e* · †*a*E · ‡*ae*;



there is not even one word of the third set, but there is at least one word in each of the other three sets.

v. ‡†*ae*, "all present but *ae*," || ‡AE · ‡A*e* · ‡*a*E · †*ae*;

{ A————————…………………… there is not even one word of the
{ E…………………———————— fourth set, but there is at least one word in each of the other three sets.

vi. ††AE, "one absent with AE," also written ††*ae*, "one absent with *ae*," (because AE and *ae* can be absent together, and neither A*e* nor *a*E can be absent at the same time with either AE or *ae*, consistently with the "general condition,") || †AE · ‡A*e* · ‡*a*E · †*ae*;

{ A————————…………………… there is not even one word of the
{ E……………………———————— first and fourth sets, but there is at least one word in the second and also in the third set. This is the arrangement of *complete diversity*, for (1) no A-words are E-words, (2) no E-words are A-words, (3) no *a*-words are *e*-words, and (4) no *e*-words are *a*-words.

vii. ††A*e*, "one absent with A*e*," also written ††*a*E, "one absent with *a*E," (because A*e* and *a*E can be absent together, and neither AE nor *ae* can be absent at the same time with either A*e* or *a*E, consistently with the "general condition,") || ‡AE · †A*e* · †*a*E · ‡*ae*;

{ A————————…………………… there is not even one word of the
{ E————————…………………… second and third sets, but there is least one word of the first and also of the fourth set. This is the arrangement of *complete identity*, for (1) every one of the A-words is an E-word, (2) every one of the E-words is an A-word, (3) every one of the *a*-words is a *e*-word, and (4) every one of the *e*-words is a *a*-word. Hence, so far as the words thought of are concerned, it would be indifferent whether we said any given word were an A-word or an E-word, a *a*-word or a *e*-word. But the possession of an A is a very different mark from the possession of an E. Hence ††A*e* or ††*a*E points to identical groups formed by means of different marks A and E. This is really a point of great importance.

These seven arrangements are the only arrangements that can possibly occur consistently with the general condition. This fact should be made perfectly clear to the mind of the pupils, and not a step further should be taken until every pupil is familiar with it, and has tested it in every possible way, because it is the fundamental fact of all deduction.

14. Now make the following observations, leading to—

The Eight Usual Arrangements (*or Incomplete Assertions*).

i. ‡AE (read strictly according to the interpretation given in art. 12) occurs in 5 of the 7 arrangements, namely, 1, 3, 4, 5, 7, and does *not* occur in 2, 6. That is, if I know that one of the pupils has an AE-set, I know that he must have one of those five arrangements; but, not knowing which of the five, I can't tell where he has an A*e*-set (present in 1, 4, 5, but absent in 3, 7) or an *a*E-set (present in 1, 3, 5, but absent in 4, 7), or an *ae*-set (present in 1, 3, 4, 7, but absent in 5). The limited knowledge, ‡AE, consequently leaves the presence or absence of each of the other three sets of words, A*e*, *a*E, *ae*, in doubt, but not in unlimited doubt. This we gather also from the four limitates given in art. 12. Thus ‡AE in conjunction with the two ‡^{1}(AE, A*e*) and ‡^{1}(AE, *a*E), leaves A*e*, *a*E in unlimited doubt. But the two other limitates ‡^{1}(A*e*, *ae*) and ‡^{1}(*a*E, *ae*), limit this doubt, by showing that there are only five possible cases of the presence and absence of A*e*, *a*E, *ae*, which are easily deduced, and seen to be the same as those pointed out by the seven complete arrangements. This is very easily done with the counters and then registered, and every pupil should be made to do it for himself, and write out his process at length. It is an important and very easy exercise.

Hence ‡AE implies ‡^{1}(A*e*, *ae*) · ‡^{1}(*a*E, *ae*).

{ A————————…………………… The diagram will have an incom-
{ E———— plete line for E, and this line can be filled up so as to make any one of the diagrams 1, 3, 4, 5, 7, and hence satisfy the two limitates, but can *not* be filled up so as to form either of the diagrams 2, 6. The teacher must see that the pupil actually fills up the diagram in all these ways for himself, and feels that there are no other ways in which it can be filled up, and so on in each of the following cases.

ii. ‡A*e* occurs in the five arrangements 1, 2, 4, 5, 6, and does *not* occur in 3, 7. And by the same process as before we see that ‡A*e* implies ‡^{1}(AE, *a*E) · ‡^{1}(*a*E, *ae*), and may be represented thus:

{ A————————…………………… which may be filled up into the
{ E ……… diagrams of 1, 2, 4, 5, 6, only.

iii. ‡*a*E occurs in the five arrangements 1, 2, 3, 5, 6, and does *not* occur in 4, 7; so that ‡*a*E implies ‡^{1}(AE, A*e*) · ‡^{1}(A*e*, *ae*), and may be represented thus:

{ A————————…………………… which may be filled up into the
{ E ———— diagrams of 1, 2, 3, 5, 6, only.

iv. ‡*ae* occurs in the five arrangements 1, 2, 3, 4, 7, and does *not* occur in 5, 6; so that ‡*ae* implies ‡^{1}(AE, A*e*) · ‡^{1}(AE, *a*E), and may be represented thus:

{ A————————…………………… which may be filled up into the
{ E ……… diagrams of 1, 2, 3, 4, 7, only.

v. †AE occurs in the two arrangements 2, 6, and does *not* occur in 1, 3, 4, 5, 7, so that †AE implies ‡A*e* · ‡*a*E, but gives ?*ae*, where the doubt is absolutely unlimited, and may be represented thus:

{ A————————…………………… which may be filled up into the
{ E…………………———— diagrams 2 or 6, but into no other. Hence we see that (1) *not even one* of the words within the range of thought that contains A also contains E, but that (2) *every one* of the words containing A is without E, (3) *every one* of the words containing E is without A, and (4) it is doubtful whether any one word without A will also be without E, and any one word without E will also be without A.

All these conclusions to be drawn from †AE should be made perfectly familiar. It should also become manifest that †AE is implied by any one of the first three of these four assertions (in conjunction with the "general condition"), but that the fourth assertion makes no addition to our knowledge, because it would be applicable to any one of the seven arrangements, one of which we know must happen, even when no statement at all has been made.

vi. †A*e* occurs in the two arrangements 3, 7, and does not occur in 1, 2, 4, 5, 6, so that †A*e* implies ‡AE · ‡*ae*, but gives ?*a*E, and may be represented thus:

{A——————………………
{E————   ………

which may be filled up into the diagrams 3 or 7, but into no other. Hence we see that (1) *not even one* of the words within the range of thought, that contains A is without E, but that (2) *every one* of the words containing A also contains E, and (3) *every one* of the words without E is also without A, and (4) it is doubtful whether any one word without A will have an E, and whether any one word which has an E will be without A. All these conclusions from †A*e* should also be quite familiar, as also that †A*e* is the one sufficient representation of the first three of these four assertions, and that the fourth assertion makes no addition to our knowledge.

vii. †*a*E occurs in the two arrangements 4, 7, and does not occur in 1, 2, 3, 5, 6, so that †*a*E implies ‡AE · *ae*, but gives ?A*e*, and may be represented thus:

{A——————………………
{E———   ………………

which may be filled up into diagrams 4 and 7, but into no other. Hence we see that (1) *not even one* of the words within the range of thought, that contains E is without A, but that (2) *every one* of the words containing E also contains A, and (3) *every one* of the words without A is also without E, and (4) it is doubtful whether any one word which has an A will be without E, or any one word without E will have an A. Also observe that †*a*E is the one sufficient representation of the first three of these four assertions, and that the fourth assertion makes no addition to our knowledge.

viii. †*ae* occurs in the two arrangements 5, 6, and does not occur in 1, 2, 3, 4, 7, so that †*ae* implies ‡A*e* · ‡*a*E, but gives ?AE, and may be represented thus:

{A——————………………
{E   ………——————

which may be filled up into diagrams 5 and 6, but into no other. Hence we see that (1) *not even one* of the words within the range of thought which is without A is without E, or which is without E is without A, but that (2) *every one* of the words without A contains E, and (3) *every one* of the words without E contains A, and (4) it is doubtful whether any word containing A contains E, or any word containing E contains A. Also observe that †*ae* is the one sufficient representation of

the first three of these four assertions, and that the fourth assertion makes no addition to our knowledge.*

15. Let the teacher now draw the pupil's attention (by questions) to the facts (1) that no set of words can be both present and absent, (2) that every one of the 7 arrangements asserts the presence (or absence) of some set of which the absence (or presence respectively) is asserted in every one of the other arrangements, and hence (3) that no two of the 7 arrangements can hold good for the same original selection of words or range of thought, and (4) that this fact is expressed by saying that assertions which imply that any two of the arrangements hold at the same time are *inconsistent*, so that one or the other of them must be *false*, but (5) only the arrangements of 6 and 7 are *totally* inconsistent, or *contradictory*, because of only these two can it be said that one asserts the presence of every set of which the other asserts the absence.

As regards the 8 incomplete (in relation to the 7 complete) arrangements, any one of the first 4, (‡AE, ‡A*e*, ‡*a*E, ‡*ae*,) is, as appears in art. 14, consistent with 5, and inconsistent with 2, of the 7 complete arrangements, and any one of the last 4, (†AE, †A*e*,

* There are 11 other incomplete arrangements, of a complex character, which are here added for the convenience of the teacher:—

i.	‡(A)	‖ ‡AE · ‡A*e* · ‡[1] (*a*E, *ae*).	{A ———— {E————…………………
ii.	‡(E)	‖ ‡AE · ‡*a*E · ‡[1] (A*e*, *ae*).	{A………—— {E————…………………
iii.	‡(*a*)	‖ ‡*a*E · ‡*ae* · ‡[1] (AE, A*e*).	{A ……………… {E————…………………
iv.	‡(*e*)	‖ ‡A*e* · ‡*ae* · ‡[1] (AE, *a*E).	{A ——……… {E————…………………
v.	‡(AE, *ae*)	‖ ‡AE · ?A*e* · ?*a*E · ‡*ae*.	{A ——……… {E————…………………
vi.	‡(A*e*, *a*E)	‖ ?AE · ‡A*e* · ‡*a*E · ?*ae*.	{A ………—— {E————…………………
vii.	‡(A, E)	‖ ‡AE · ‡A*e* · ‡*a*E · ?*ae*.	{A………———— {E————…………………
viii.	‡(A, *e*)	‖ ‡AE · ‡A*e* · ?*a*E · ‡*ae*.	{A ————……… {E————…………………
ix.	‡(*a*, E)	‖ ‡AE · ?A*e* · ‡*a*E · ‡*ae*.	{A——……………… {E————…………………
x.	‡(*a*, *e*)	‖ ?AE · ‡A*e* · ‡*a*E · ‡*ae*.	{A ………——……… {E————…………………
xi.	?|AE|	‖ ‡[1] (AE, A*e*) · ‡[1] (*a*E, *ae*) · ‡[1] (AE, *a*E) . ‡[1] (A*e*, *ae*).	{A—— ……… {E —— ………

These 26 assertions (7 complete, 8 simple incomplete, 11 complex incomplete) are all that can be made respecting two letters (or names). The importance of this complete enumeration will appear in art. 30, note.

†*a*E, †*ae*,) is consistent with 2, and inconsistent with 5, of the same. This consistency arises from the incompleteness of these assertions, by virtue of which all the inconsistent portions are left out of consideration. The teacher will observe for subsequent use, the bearing of this last remark upon all cases of general assertions, and on the distinction between abstract and concrete. It is only by dwelling on points of known resemblance, and passing over points of known or possible dissimilarity, that we are able to arrive at the notion of general laws, or invariable relations apprehended in the midst of infinite variability.

Lastly, comparing the 8 incomplete arrangements with each other only, observe that any one of the first four, (‡AE, ‡A*e*, ‡*a*E, ‡*ae*,) is consistent with 6, and inconsistent with only one of the rest. Thus ‡AE is consistent with ‡A*e*, ‡*a*E, ‡*ae*, †A*e*, †*a*E, †*ae*, and inconsistent with †AE alone. Also, any one of the last four, (†AE, †A*e*, †*a*E, †*ae*,) is consistent with 4, and inconsistent with 3, of the remainder, (admitting the general condition). Thus †AE is consistent with ‡A*e*, ‡*a*E, (both of which it implies,) and also with ‡*ae*, †*ae*, (neither of which it implies,) and inconsistent with ‡AE, †A*e*, †*a*E. Hence we disprove ‡AE, by proving †AE, and we disprove †AE by proving ‡AE, or else either †A*e* or †*a*E, admitting the general conditions; for if they are not admitted, so that we might have †A or †E, there would be nothing to prevent †AE coexisting with either †A*e* or †*a*E. But there is this difference between these disproofs, that ‡AE disproves †AE directly, without reference to the general condition; and either †A*e* or †*a*E disproves †AE by implication only, in consequence of the general condition, and this distinction has been recognized in the old logic, which did not explicitly introduce the general condition. Thus, according to the old logic, †AE and ‡AE, or †A*e* and ‡A*e*, or †*a*E and ‡*a*E are *contradictories*, but †AE and †A*e*, or †AE and †*a*E, are *contraries*. The old logic also calls ‡AE and ‡A*e* or ‡AE and ‡*a*E, *subcontraries*, the real relations being, as we have already seen ‡^{1}(AE, A*e*) and ‡^{1}(AE, *a*E).

Observe that ‡AE and †AE, if the general condition be admitted, do not affirm and deny the existence of precisely the same sets of words, but merely of one particular set of AE-words. Written at full, they are ‡AE · ‡^{1}(A*e*, *ae*) · ‡^{1}(*a*E, *ae*), and †AE · ‡A*e* · ‡*a*E · ?*ae*; whence it appears at once that †AE is perfectly consistent with 2 out of the 5 arrangements of ‡^{1}(A*e*, *ae*) · ‡^{1}(*a*E, *ae*), namely, either ‡A*e* · ‡*a*E · ‡*ae* or ‡A*e* · ‡*a*E · †*ae*, but not with the 3 remaining arrangements, which involve either †A*e* or †*a*E, or both †A*e*, and †*a*E.

A clear comprehension of these relations of consistency and inconsistency is of great value in actual reasoning, and hence it is worth the teacher's while to dwell on them. However perplexing they may be as abstract propositions, they become simplicity itself when referred to the concrete case of words grouped according to the vowels A and E.

16. The above observations are all so simple, and evident when made upon the letters forming words in the manner suggested

that they may appear merely play to the child, and the teacher may run the risk of hurrying over them too fast. Let me impress upon any one who adopts the method I am explaining, to guard against any such error. Every particle of what has here been suggested is of extreme importance afterwards, and a correct general view of the usual logical relations and assertions cannot be obtained without a perfect mastery of such apparently self-evident relations, which can be best acquired by confining attention to one simple but sufficient class of phenomena, such as the vowels contained in English words; and although the step to general propositions referring to matters usually spoken of, might now be easily taken, I believe it will be best to postpone their consideration until a certain amount of dexterity of manipulation has been acquired in the use of the preceding symbols or counters, by proceeding to syllogisms, still solely relating to the presence or absence of certain vowels in a certain set of English words chosen at random, or subject to certain conditions of presence and absence. This is the course which I proceed to explain.

17. Consider words in respect to three vowels, A, E, I. First form AE, A*e*, *a*E, *ae* as before. Then words containing both A and E, must contain I or not, and hence if ‡AE, there must be ‡^{1}(AEI, AE*i*). Thus *increase* is an AEI-word, but *please* is an AE*i*-word. Similarly if ‡A*e*, then ‡^{1}(A*e*I, A*ei*.) Also if ‡*a*E, then ‡^{1}(*a*EI, *a*E*i*), and if ‡*ae* then ‡^{1}(*ae*I, *aei*). Now, when we do not know what words have been selected, we only know that all or some of the four compounds AE, A*e*, *a*E, *ae* may be present, and hence that all or some of the eight compounds AEI, AE*i*, A*e*I, A*ei*, *a*EI, *a*E*i*, *ae*I, *aei* may be present; and *no others*. Now begin with EI, E*i*, *e*I, *ei*, and show that if ‡EI, then ‡^{1}(AEI, *a*EI) and so on. The same eight compounds will result and no others. Again begin with AI, A*i*, *a*I, *ai*, and show that if ‡AI, then ‡^{1}(AIE, AI*e*) and so on. The same eight compounds again result. All this is well shown by counters marked with these compounds, thus—

AE	A*e*	*a*E	*ae*
AEI, AE*i*	A*e*I, A*ei*	*a*EI, *a*E*i*	*ae*I, *aei*

And then altering the top line, show that you have only to shift the counters thus:

AI	A*i*	*a*I	*ai*
AEI, A*e*I	AE*i*, A*ei*	*a*EI, *ae*I	*a*E*i*, *aei*

Or thus:

EI	E*i*	*e*I	*ei*
AEI, *a*EI	AE*i*, *a*E*i*	A*e*I, *ae*I	A*ei*, *aei*

18. Then observe that we don't know until we are told whether any of the smaller (or inferior) compounds, AE, A*e*, *a*E, *ae*, or AI, A*i*, *a*I, *ai*, or EI, E*i*, *e*I, *ei* are absent or present. But if any one *is present*, then *at least one of the two larger (or superior) compounds* under it in these forms must be present, and both may be; but if any one *is absent, both of the corresponding larger compounds must be absent*. Thus if we know that there is no word containing E without I, or †E*i*, then we know that there is no word containing

A and E without I, or containing E without either A or I. That is, if †E*i* then †(AE*i*, *a*E*i*). But we know that if †E*i* then ‡EI · ‡*ei*·?*e*I, hence we know ‡^{1}(AEI, *a*EI) · ‡^{1} (A*ei*, *aei*) · ? (A*e*I, *ae*I). All these consequences of †E*i* should be very distinctly seized and well exemplified by words. Get the whole class to write down twenty words a-piece, each having 2 or 3 vowels, and none having E without I, taking care that the general condition is fulfilled; ascertain that the above results are obtained, by actually sorting first with respect to E, *e*; then each bundle with respect to I, *i*; and finally each with respect to A, *a*. Some may have ‡A*e*I, others †A*e*I; some may have ‡*ae*I, others †*ae*I. None will have ‡AE*i* or ‡*a*E*i*. Some may have ‡AEI·‡*a*EI, others ‡AEI·†*a*EI, others †AEI · ‡*a*EI, but none †AEI · †*a*EI, and so on. By this means the real meaning of the signs and processes will become evident to them, and they will feel that they are dealing with a law of thought.

Any set of words being given, the pupils should be exercised in marking what compounds are present or absent. Thus *avail, Italy, debasement, conceive, seem,* give †AEI · ‡AE*i* · ‡A*e*I · †A*ei* · ‡*a*EI ·‡*a*E*i* · †*ae*I · †*aei*. This is best done by writing the compounds on the black board, and writing against them each word as it arises, thus,—

AEI		*a*EI	conceive
AE*i*	debasement	*a*E*i*	seem
A*e*I	avail, Italy	*ae*I	
A*ei*		*aei*.	

All the words being used up, the blanks show the absent sets. This exercise is really important.

19. Next suppose we happened to know which of the larger compounds were present or absent or doubtful, see if we could find out what is the case with the smaller.

It is quite clear that if ‡AEI, then ‡AE, ‡AI, and ‡EI. Thus *increase* being an AEI-word, is also an AE-word, and an AI-word, and an EI-word; so that if there is at least one such word as the first, there is by that means at least one such word as each of the three last.

Again, if ‡^{1}(AEI, AE*i*), then certainly ‡AE; because in that case either ‡AEI, or ‡AE*i*, or both, and in either case, as we have seen, there must be ‡AE. Thus if we know that either *increase* or *mute*, or both, are among the words thought of, we know that there must be at least one AE-word.

If ‡AEI · †AE*i*, or if ‡AEI· ?AE*i*, still there must be ‡AE on account of ‡AEI.

If †AEI · †AE*i*, then, and in no other case, †AE, That is, if no word thought of contains both A and E, either with or without I, then no word at all contains both A and E.

If ?AEI· ?AE*i*, or if ?AEI·†AE*i*, or if †AEI· ?AE*i*, then ?AE. For if all the ? were †, then †AE, by the last case; but if only one of them were ‡, then ‡AE, by the former cases. But we do not know which is the case, hence ?AE.

If three compounds all containing the same letter are †, then the fourth will be ‡. Thus if †AEI ·†AE*i* ·†A*e*I, then ‡A*ei*. Be-

cause by the general condition there must be at least one compound present containing A, and only these four compounds can contain A. This is often very useful.

All these cases must be worked out over and over again, with all varieties, and for any two or three of the 8 larger compounds, and results deduced with ease and certainty, just as in simple arithmetic.

Where no doubtful compounds occur, the pupils should give words completely illustrating given cases. Where doubtful compounds occur, they should give sets of words answering to each case included in the doubt, not omitting any one included, but not giving any one excluded.

20. But suppose the general condition is not fulfilled, what happens? Suppose there is at least one word containing A, but no *a*-word, that is no word without A. Then of course all the words are A words, and we have †*a*E and †*ae* and hence †*a*EI · †*a*E*i* and †*ae*I · †*aei*. This occasions no difficulty. But it will be seen that the only compounds which can be present are AEI, AE*i*, A*e*I, A*ei*, all of which contain A, and are in fact the compounds EI, E*i*, *e*I, *ei*, with A prefixed. Hence there is really no use in mentioning A at all. The range of thought is confined to A-words.

But suppose there is *only one* word containing A, what happens? One of the two compounds AE, A*e* must be present, and *only one can be* present. In counters show this by grouping $\boxed{AE \mid Ae}$ *not* inverted. In writing prefix ‡$_1$, with the little 1 below, thus, ‡$_1$(AE, A*e*), and read "present only one of AE, A*e*." We have then also ‡$_1$ (AEI, AE*i*, A*e*I, A*ei*). In writing, it is often convenient to put a little $_1$ below the letter which occurs in only one word. Thus ‡A$_1$ E shows that there is only one word containing A, and that that word also contains E, so that †A$_1$*e*. This is a case of common occurrence in practice, when general propositions are used, and should be carefully studied in every case. A little rider of paper, or a clip, placed across the letter on the counter, or an elastic band strung round it, will sufficiently mark A$_1$, &c., as the case may be.

21. Every preparation has now been made for considering syllogisms, and indeed much more complicated cases. Considered as problems upon words containing letters, the syllogism is as follows:—*Given the state of a certain group of words with respect to containing A and E; and also their state with respect to containing E and I; to find their state with respect to containing A and I.* These states are given by one of the complete or incomplete arrangements of Art. 13 or 14. The two first are called the *premisses*, and the last the *conclusion*. The letters A and I are called the extremes. The letter E, which occurs in both the premisses, but does not occur in the conclusion, is called the *mean*. The process is as follows, and may be conducted with counters or on paper, but for children counters are far best.

First, state the two premisses, in the abbreviated form of Arts. 13 and 14, forming the upper line of counters, remembering all the implied present, absent, or doubtful parts.

A and E without I, or containing E without either A or I. That is, if †E*i* then †(AE*i*, *a*E*i*). But we know that if †E*i* then ‡EI · ‡*ei*·?*e*I, hence we know ‡^{1}(AEI, *a*EI) · ‡^{1} (A*ei*, *aei*)·?(A*e*I, *ae*I). All these consequences of †E*i* should be very distinctly seized and well exemplified by words. Get the whole class to write down twenty words a-piece, each having 2 or 3 vowels, and none having E without I, taking care that the general condition is fulfilled; ascertain that the above results are obtained, by actually sorting first with respect to E, *e*; then each bundle with respect to I, *i*; and finally each with respect to A, *a*. Some may have ‡A*e*I, others †A*e*I; some may have ‡*ae*I, others †*ae*I. None will have ‡AE*i* or ‡*a*E*i*. Some may have ‡AEI·‡*a*EI, others ‡AEI·†*a*EI, others †AEI · ‡*a*EI, but none †AEI · †*a*EI, and so on. By this means the real meaning of the signs and processes will become evident to them, and they will feel that they are dealing with a law of thought.

Any set of words being given, the pupils should be exercised in marking what compounds are present or absent. Thus *avail*, *Italy*, *debasement*, *conceive*, *seem*, give †AEI · ‡AE*i* · ‡A*e*I · †A*ei* · ‡*a*EI ·‡*a*E*i* · †*ae*I · †*aei*. This is best done by writing the compounds on the black board, and writing against them each word as it arises, thus,—

AEI		*a*EI	conceive
AE*i*	debasement	*a*E*i*	seem
A*e*I	avail, Italy	*ae*I	
A*ei*		*aei*.	

All the words being used up, the blanks show the absent sets. This exercise is really important.

19. Next suppose we happened to know which of the larger compounds were present or absent or doubtful, see if we could find out what is the case with the smaller.

It is quite clear that if ‡AEI, then ‡AE, ‡AI, and ‡EI. Thus *increase* being an AEI-word, is also an AE-word, and an AI-word, and an EI-word; so that if there is at least one such word as the first, there is by that means at least one such word as each of the three last.

Again, if ‡^{1}(AEI, AE*i*), then certainly ‡AE; because in that case either ‡AEI, or ‡AE*i*, or both, and in either case, as we have seen, there must be ‡AE. Thus if we know that either *increase* or *mate*, or both, are among the words thought of, we know that there must be at least one AE-word.

If ‡AEI · †AE*i*, or if ‡AEI· ?AE*i*, still there must be ‡AE on account of ‡AEI.

If †AEI · †AE*i*, then, and in no other case, †AE, That is, if no word thought of contains both A and E, either with or without I, then no word at all contains both A and E.

If ?AEI· ?AE*i*, or if ?AEI·†AE*i*, or if †AEI· ?AE*i*, then ?AE. For if all the ? were †, then †AE, by the last case; but if only one of them were ‡, then ‡AE, by the former cases. But we do not know which is the case, hence ?AE.

If three compounds all containing the same letter are †, then the fourth will be ‡. Thus if †AEI ·†AE*i* ·†A*e*I, then ‡A*ei*. Be-

cause by the general condition there must be at least one compound present containing A, and only these four compounds can contain A. This is often very useful.

All these cases must be worked out over and over again, with all varieties, and for any two or three of the 8 larger compounds, and results deduced with ease and certainty, just as in simple arithmetic.

Where no doubtful compounds occur, the pupils should give words completely illustrating given cases. Where doubtful compounds occur, they should give sets of words answering to each case included in the doubt, not omitting any one included, but not giving any one excluded.

20. But suppose the general condition is not fulfilled, what happens? Suppose there is at least one word containing A, but no *a*-word, that is no word without A. Then of course all the words are A words, and we have †*a*E and †*ae* and hence †*a*EI · †*a*E*i* and †*ae*I · †*aei*. This occasions no difficulty. But it will be seen that the only compounds which can be present are AEI, AE*i*, A*e*I, A*ei*, all of which contain A, and are in fact the compounds EI, E*i*, *e*I, *ei*, with A prefixed. Hence there is really no use in mentioning A at all. The range of thought is confined to A-words.

But suppose there is *only one* word containing A, what happens? One of the two compounds AE, A*e* must be present, and *only one can be* present. In counters show this by grouping $\boxed{\text{AE} \mid \text{A}e}$ *not* inverted. In writing prefix ‡$_1$, with the little 1 below, thus, ‡$_1$(AE, A*e*), and read "present only one of AE, A*e*." We have then also ‡$_1$ (AEI, AE*i*, A*e*I, A*ei*). In writing, it is often convenient to put a little $_1$ below the letter which occurs in only one word. Thus ‡A$_1$ E shows that there is only one word containing A, and that that word also contains E, so that †A$_1$*e*. This is a case of common occurrence in practice, when general propositions are used, and should be carefully studied in every case. A little rider of paper, or a clip, placed across the letter on the counter, or an elastic band strung round it, will sufficiently mark A$_1$, &c., as the case may be.

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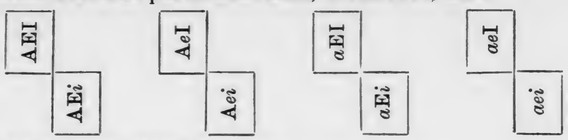
First, state the two premisses, in the abbreviated form of Arts. 13 and 14, forming the upper line of counters, remembering all the implied present, absent, or doubtful parts.

Secondly, arrange the 8 compounds below.

Thirdly, work with one premiss upon those compounds, as shown by art. 18, and then with the other upon the result. The final result is called the *resultant*.

Fourthly, from this resultant, deduce the conclusion as in art. 19.

Fifthly, place this conclusion in the top line. The examples will make this quite easy. Every case given should be worked over and over again, to gain ease, rapidity, and certainty. Be sure not to hurry. If properly conducted, the work will be found very amusing. For convenience of printing, only written signs are given. For ‡AEI the teacher will show black side of counter erect; for ?AEI, black side inverted; and for †AEI, red side. Before being thus arranged so as to indicate that any one is present, absent, or doubtful, the eight counters of the larger compounds will be all put on their sides, in two rows, thus—



This may be indicated by writing

(AEI)	(AeI)	(aEI)	(aeI)
(AEi)	(Aei)	(aEi)	(aei)

22. *First kind of Syllogism.*

"No words have A without E or E without I; then no words have A without I."

First premiss. "No words have A without E," or †Ae, or "all words having A have E." This implies ‡AE · ‡ae and ?aE.

Second premiss. "No words have E without I," or †Ei, or "all words having E have I." This implies ‡EI · ‡ei and ?eI.

Resultant. First work for †Ae, and get †AeI and †Aei. Then work for †Ei, and get †AEi and †aEi. Hence, in counters, the resultant will stand thus—

(AEI)	†AeI	(aEI)	(aeI)
†AEi	†Aei	†aEi	(aei)

Rule: whenever there are † compounds in the premisses, work with them first.

Next go to the implied ? compounds, and work first with ?aE. This gives only ?aEI, for †aEi is already determined. Next work with ?eI, and get ?aeI, since †AeI is already determined. The resultant now stands in counters—

(AEI)	†AeI	?aEI	?aeI
†AEi	†Aei	†aEi	(aei)

Next work for the ‡ compounds of the premisses. Then ‡AE gives ‡'(AEI, AEi), but we have already got †AEi, hence we must have ‡AEI, in order that one of the two compounds should be present. Next try ‡ae, giving ‡'(aeI, aei). But here we have ?aeI, and hence also ?aei, so far as this is concerned. But we have

still the ‡ compounds of the other premiss to try. Now ‡EI gives ‡'(AEI, aEI), and as we have already ‡AEI, we have ?aEI, so that we are not advanced. Next ‡ei gives ‡'(Aei, aei), and here since †Aei, we must have ‡aei, so that the previous ?aei becomes ‡aei.

The final form of the resultant is, therefore,

‡AEI	†AeI	?aEI	?aeI
†AEi	†Aei	†aEi	‡aei

The two ‡ compounds might have been found from the fact that they are the only non-absent compounds containing A and *i* respectively.

Now make a diagram of this syllogism, thus—

```
{ A ——             ..................
{ E ——————————..................
{ I ————————————          ......
```

and show how it agrees with the resultant.

Conclusion. ‡AEI · †AEi give ‡AI; †AEi · †Aei give †Ai; ?aEI · ?aeI give ?aI; and †aEi · ‡aei give ‡ai. Hence the conclusion is †Ai. Now place the counter †Ai on the upper ledge, widely separated from the premisses. In writing put ∴, read "therefore," before the conclusion. This conclusion shows that there are no words having A without I.

Write the whole syllogism thus: †Ae · †Ei ∴ †Ai.

Illustrate by words. Get each pupil to do it separately. Probably the ? compounds will be differently determined by different pupils, as in the four following instances.

i. *beating, measuring, seeming, sin, fog.* This gives ‡†Ae and ‡†Ei as premisses, and ‡†Ai as conclusion. Show that in such a case instead of ?aEI, and ?aeI, we *must* have (as in the particular instance selected) ‡aEI · ‡aeI. Compare art. 30, note, 14.

ii. *beating, measuring, sin, fog.* This gives ††Ae and ‡†Ei, with ‡†Ai as conclusion again, but ?aEI and ?aeI are now †aEI and ‡aeI. Compare art. 30, note, 16.

iii. *beating, measuring, seeming, fog.* This gives ‡†Ae and ††Ei, with the conclusion ‡†Ai, as in the preceding cases, but now ?aEI and ?aeI become ‡aEI and †aeI. Compare art. 30, note, 16.

iv. *beating, measuring, fog.* This gives ††Ae and ††Ei with the conclusion ††Ai, which differs from all the preceding, and now ?aEI and ?aeI become †aEI and †aeI. Compare art. 30, note, 17.

Observe that in working out each separately, it will be best to display the premisses and conclusion in counters, thus—

(i.)	†Ae	‡aE	†Ei	‡eI	†Ai	‡aI
(ii.)	†Ae	†aE	†Ei	‡eI	†Ai	‡aI
(iii.)	†Ae	‡aE	†Ei	†eI	†Ai	‡aI
(iv.)	†Ae	†aE	†Ei	†eI	†Ai	†aI

The pupils should see that it is only the ? compounds that are affected by these changes, and that, although the three first conclusions are the same, the corresponding resultants (which alone fully represent the premisses) are different. All the different cases must always be furnished with instances in words.

23. There are eight different cases of this first kind of syllogism, namely—

†AE·†eI ∴ †AI	†aE·†eI ∴ †aI
†AE·†ei ∴ †Ai	†aE·†ei ∴ †ai
†Ae·†EI ∴ †AI	†ae·†EI ∴ †aI
†Ae·†Ei ∴ †Ai	†ae·†Ei ∴ †ai

These are distinguished by having the short forms of the premisses both †, and the mean E in one and e in the other, that is, the contrary in one to what it is in the other. This gives the "skeleton rule."

[1.] *Means unlike. Both premisses †. Conclusion, † extremes.*

This is a valuable rule. See how it applies in each case. Each case has four subcases as before, and each should be worked out separately, with counters, with diagrams, and with words.

The pupils should be exercised in drawing diagrams for each of these syllogisms, and in showing all the four cases which can arise from filling up the blanks in each. In the diagram given above the E-line, or line of the mean, has been completely filled, and the doubts thrown on the other lines. The teacher should vary this, and much vary the lengths of the lines, and show what the essential relations are. This will be excellent practice in understanding the effect of combined assertions.

It will soon be found that it is possible to introduce into the diagram a condition which is not contained in the premisses, although the full incompleteness of the premisses is retained. Thus the diagram—

A————————..................
E——————   ......
I —————————......———

satisfies ‡AE·†Ae·?aE·‡ae, and also ‡EI·†Ei·?eI·‡ei. But it also manifestly gives ‡AI·†Ai·‡aI·‡ai or ‡†Ai instead of †Ai, as a conclusion. Why so? Because a new relation has been furtively introduced, limiting the absolute doubtfulness attachable to ?aE and ?eI, by rendering it impossible for them both to be absent at once, namely, ‡^{1}(aE, eI), which gives in the resultant ‡^{1}(aEI, aEi, AeI, aeI), or since †aEi·†AeI, only ‡^{1}(aEI, aeI), whence ‡aI. Thus it is impossible to fill up the blanks in the E line without introducing ‡aEI, or ‡aeI, or both. In fact, both the A and I lines are completely filled, and the doubts are thrown on the single line E. It will be readily seen that we cannot avoid introducing such an additional condition, if we begin by filling up the line of either of the extremes instead of that of the mean, as in this case we should have also to fill up the line of the other extreme completely. But if the lines of both extremes are filled up, the conclusion appears as a complete, instead of an incomplete arrangement, limiting the absolute doubtfulness of the two ? compounds in the resultant. In actual argument, such new conditions are often covertly introduced, even without the knowledge of the arguer, especially when he is exemplifying his argument, and are then very difficult to detect. But it is always necessary to detect them, as the new condition alters the nature of the argument. In the present case, this new condition

removes the argument from the class of syllogisms, because it introduces a third premiss, namely ‡^{1}(aE, eI), which relates to all three of the letters A, E, I at once.

24. *The Second kind of Syllogism.*

"If no words have E without A, and also no words have E without I; then at least one word has both A and I."

First premiss. No words have E without A, or †aE.
Second premiss. No words have E without I, or †Ei.
Resultant. After using †aE, †Ei,

(AEI)	(AeI)	†aEI	(aeI)
†AEi	(Aei)	†aEi	(aei)

In this case only *three* compounds are †.

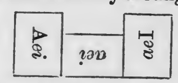
Next work with ?Ae and ?eI, and the resultant becomes

(AEI)	?AeI	†aEI	?aeI
†AEi	?Aei	†aEi	(aei)

Now take ‡AE, which gives ‡^{1} (AEI, AEi), and hence, since †AEi, determines ‡AEI. But ‡ae gives only ‡^{1}(aeI, aei), and this is the utmost limitation we can get. Next ‡EI gives ‡^{1}(AEI, aEI), and as †aEI, we have ‡AEI as before. Lastly, ‡ei gives ‡^{1}(Aei, aei) and this is the utmost limitation we can get. Now observe that the general conditions are satisfied, without taking any consideration of the compound AeI, which was left quite doubtful by the premisses. The final resultant is

‡AEI	?AeI	†aEI	‡^{1}(aeI, aei)·‡^{1}Aei, aei)
†AEi		†aEi	

The two *limitates* are indicated by arranging the counters thus—



Conclusion. ‡AI from ‡AEI; ‡^{1}(Ai, ai) from ‡^{1}(Aei, aei); and ‡^{1}(aI, ai) from ‡^{1}(aeI, aei). Observe that ?AeI would give ?AI, but as we have already found ‡AI from ‡AEI, this does not affect the conclusion. The only *certain* thing, then, is, ‡AI, or that at least one word has both letters A and I.

Write the whole syllogism thus †aE·†Ei ∴ ‡AI, the two limitates of the conclusion being implied, as in art. 14.

This is rather a difficult case, especially for the diagram

A ————————   ......
E ————————..................
I ————————   ......

This must also be furnished with words, and the four subcases worked out, as in the first kind of syllogism. The results being

‡†aE·‡†Ei ∴ ‡AI	††Ae·‡†Ei ∴ ‡†Ai
‡†aE·††Ei ∴ ‡†aI	††Ae·††Ei ∴ ††Ai.

Be very particular in obtaining the full resultant for each case. Show how the diagram and the first resultant can be altered for each particular case without disturbing any of the fixed parts.

The first case may present a little difficulty, because the pupil will be apt to consider, that when the assertions are complete, he

has to fill up the whole lines, and leave no empty parts. We are ignorant of the extent to which the lines should overlap, and the diagram for ‡†*a*E · ‡†E*i* should present so many blanks that it can be filled up into any one of the forms of the resultant which would give any one of the 5 forms of the conclusion ‡AI · ‡^{1}(A*i*, *ai*) · ‡^{1}(*a*I, *ai*). The following will be found to be a mode of filling up the original diagram so as to answer these conditions, but the ingenuity of the pupil should be well taxed before any solution is presented to him.

A ————————   ......
E ————————..................
I ———————   —....

25. There are also eight different cases of this second kind of syllogism, namely—

†AE · †EI ∴ ‡*ai*	†*a*E · †EI ∴ ‡A*i*
†AE · †E*i* ∴ ‡*a*I	†*a*E · †E*i* ∴ ‡AI
†A*e* · †*e*I ∴ ‡*ai*	†*ae* · †*e*I ∴ ‡A*i*
†A*e* · †*ei* ∴ ‡*a*I	†*ae* · †*ei* ∴ ‡AI.

These are distinguished by having the short forms of the premisses both † (as in art. 23), but the mean is E or *e* in *both*, (which is quite different from art. 23). This gives the "skeleton rule."

[2.] *Means like. Both premisses †. Conclusion, ‡ contraries of extremes.*

See how this applies in each case. Each case has four subcases as before, and each should be worked out separately with counters, with diagrams, and with words.

26. *The Third kind of Syllogism.*

"If at least one word has both A and E, and not one word has E without I, then at least one word has both A and I."

First premiss. At least one word has both A and E, or ‡AE.

Second premiss. Not even one word has E without I, or †E*i*.

Resultant. Working first with †E*i*, we obtain

(AEI)	(A*e*I)	(*a*EI)	(*ae*I)
†AE*i*	(A*ei*)	†*a*E*i*	(*aei*)

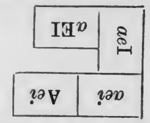
Then working for ‡^{1}(A*e*, *ae*), ‡^{1}(*a*E, *ae*), (which may be considered as ?A*e*, ?*a*E, ?*ae*, leaving the limitations to be recovered from the general conditions,) and also ?*e*I, we get—

(AEI)	?A*e*I	?*a*EI	?*ae*I
†AE*i*	?A*ei*	†*a*E*i*	?*aei*

Next work with ‡AE, which gives ‡^{1}(AEI, AE*i*); whence, since †AE*i*, we have ‡AEI, which has to be put for (AEI). Then work with ‡EI, giving ‡^{1}(AEI, *a*EI), whence, as ‡AEI has already been found, *a*EI remains doubtful. Next ‡*ei*, gives ‡^{1}(A*ei*, *aei*), and nothing further. We have now got ‡(A, E, I, *e*, *i*), but, to satisfy the general condition, we must also have ‡*a*, and this gives ‡^{1}(*a*EI, *ae*I, *aei*), leaving ?A*e*I. The complete resultant then is—

‡AEI	?A*e*I		‡^{1}(*a*EI, *ae*I, *aei*)
†AE*i*		†*a*E*i*	‡^{1}(A*ei*, *aei*)

The last limitates are most conveniently set up in counters, thus—



But when several limitates occur, and one compound enters into several of them, it is often convenient to use a special mark both in writing and with counters, called an *index*, which may be placed before each compound separately. This consists of a large number as 2 or 3, showing how many compounds there are in the limitate, and a smaller number subscribed, showing to which set of 2 or 3 the compound belongs. These should be prepared in counters, about 1½ inches wide, and 3 inches high, in sets, thus 2_1 2_1, 2_2 2_2, 2_3 2_3, 2_4 2_4, 3_1 3_1 3_1, &c.

The resultant may now be more conveniently written, thus—

‡AEI	?A*e*I	3_1*a*EI	3_1 *ae*I
†AE*i*	2_1A*ei*	†*a*E*i*	3_1 2_1 *aei*

Where 3_1 2_1 *aei* shows that *aei* belongs to two limitates. The counters may remain inverted after these indices, to show their doubtful character, but this is no longer essential, as the index itself marks *limited doubtfulness.*

Conclusion. ‡AI from ‡AEI; ‡^{1}(A*i*, *ai*) from 2_1(A*ei*, *aei*); and ‡^{1}(*a*I, *ai*) from 3_1 (*a*EI, *ae*I, *aei*). Hence the only thing certain is that at least one word contains both A and I.

Write the whole syllogism thus:

‡AE · †E*i* ∴ ‡AI

the limitates of the conclusion being implied as in art. 14.

Diagram :—

A   ——
E ————————..................
I ———————   ......

Now there are 5 different complete arrangements corresponding to the first premiss, and 2 to the second, hence there are 10 syllogisms in which the premisses would be the same in the non-doubtful parts, and each of these would lead to resultants and conclusions, also the same in the non-doubtful parts. In the present case these 10 syllogisms are—

‡†*a*E · ‡†E*i* ∴ ‡AI	‡†*ae* · ††E*i* ∴ ‡†*ai*
‡†A*e* · ‡†E*i* ∴ ‡†A*i*	††A*e* · ‡†E*i* ∴ ‡†A*i*
‡†*ae* · ‡†E*i* ∴ ‡†*ai*	††A*e* · ††E*i* ∴ ††A*i*
‡†A*e* · ††E*i* ∴ ‡†A*i*	‡‡AE · ‡†E*i* ∴ ‡AI · ‡*a*I
‡†*a*E · ††E*i* ∴ ‡†*a*I	‡‡AE · ††E*i* ∴ ‡AI · ‡A*i* · ‡*a*I · ‡*ai*.

All these cases should be worked by counters (they are worked on paper in art. 39), and exemplified, and the blanks of the diagram of the general case should be filled up so as to illustrate every one of these cases. This will be found both instructive and entertaining, and will serve to make the process of deduction something real and intelligible instead of abstract and hazy.

27. There are sixteen cases of this third kind of syllogism, eight when the first, and eight when the second premiss is ‡, namely :—

‡AE · †EI ∴ ‡A*i*	†AE · ‡EI ∴ ‡*a*I
‡AE · †E*i* ∴ ‡AI	†AE · ‡E*i* ∴ ‡*ai*
‡A*e* · †*e*I ∴ ‡A*i*	†A*e* · ‡*e*I ∴ ‡*a*I
‡A*e* · †*ei* ∴ ‡AI	†A*e* · ‡*ei* ∴ ‡*ai*
‡*a*E · †EI ∴ ‡*ai*	†*a*E · ‡EI ∴ ‡AI
‡*a*E · †E*i* ∴ ‡*a*I	†*a*E · ‡E*i* ∴ ‡A*i*
‡*ae* · †*e*I ∴ ‡*ai*	†*ae* · ‡*e*I ∴ ‡AI
‡*ae* · †*ei* ∴ ‡*a*I	†*ae* · ‡*ei* ∴ ‡A*i*.

These are distinguished by having the short form of one premiss ‡, and the other †, and having the means *like*, that is, either both E or both *e*. This gives the important "skeleton rule":—

[3]. *Means like. One premiss ‡, and one †. Conclusion, ‡ extreme of the ‡, with contrary of extreme of the † premiss.*

Thus the ‡ premiss in the first case above is ‡AE, and its extreme is A. The † premiss is †EI, and its extreme is I, of which the contrary is *i*. Hence the conclusion is ‡A*i*.

Each syllogism has 10 subcases, and each should be worked out separately with counters, with diagrams, and with words. See also art. 39.

28. *Syllogisms with Doubtful Conclusions.*

The 32 syllogisms in the three kinds just considered (with their cases), are the only syllogisms which give any certain conclusion. All others leave every compound in the conclusion either absolutely or limitedly doubtful. It is easily seen that the 16 cases in which both premisses are ‡, will leave every compound even in the resultant, doubtful. But it is not so evident that the 16 cases in which one premiss is ‡, and the other †, but *the means unlike*, will give only a doubtful conclusion, because two compounds in the resultant will always be absent. Hence it is best to work out a case.

"If at least one word has both A and E, and not one word has I without E, nothing can be predicted."

First premiss. At least one word has both A and E, or ‡AE.

Second premiss. Not even one word has I without E, or †*e*I.

Resultant. Worked for †*e*I, it becomes—

(AEI) †A*e*I (*a*EI) †*ae*I
(AE*i*) (A*ei*) (*a*E*i*) (*aei*)

Now, working for ‡AE, we get ‡¹(AEI, AE*i*), with nothing further. Working for ‡EI, we get ‡¹(AEI, *a*EI.) Working for ‡*ei*, we have ‡¹(A*ei*, *aei*). Working for ‡*a*, which is still undetermined, we get ‡¹(*a*EI, *a*E*i*, *aei*). So that, using the indices, the complete resultant is—

$2_2\ 2_1$ AEI †A*e*I $3_1\ 2_2$ *a*EI †*ae*I
2_1 AE*i* 2_3A*ei* 3_1 *a*E*i* $3_1\ 2_3$*aei*

Conclusion. For AI, either ‡¹(AI, A*i*) from 2_1 (which may be used as an abbreviation for all the compounds bearing that index) or ‡¹(AI, *a*I) from 2_2. For A*i*, either ‡¹(AI, A*i*) from 2_1 or ‡¹(A*i*, *ai*) from 2_3. For *a*I, either ‡¹(AI, *a*I) from 2_2 or ‡¹(*a*I, *ai*) from 3_1. For *ai*, either ‡¹(A*i*, *ai*) from 2_3, or ‡¹(*a*I, *ai*) from 3_1. But these four limitates ‡¹(AI, A*i*), ‡¹(*a*I, *ai*), ‡¹(AI, *a*I), ‡¹(A*i*, *ai*) result in every case from the general condition ‡A, ‡*a*, ‡E, ‡*e*, (art. 12,) and hence

add nothing to our knowledge. That is, the premisses tell us nothing as respects the possession of A and I; or the conclusion is doubtful.

Write ‡AE · †*e*I ∴ ?

Observe, however, that the resultant has a very determinate form, and that we know from †A*e*I that no words contain A and I without E, and from †*ae*I, that no words contain I without either A or E.

29. We may comprise all these 32 syllogisms under two skeleton rules, thus—

[4.] *Means unlike. One premiss ‡, and one †. Conclusion doubtful.*

[5.] *Means like or unlike. Both premisses ‡. Conclusion doubtful.*

30. The 64 cases of syllogism here considered are all admitted by Prof. De Morgan.* The old logic only admitted 11 of them,

* Teachers will perceive that the number of different syllogisms is limited by the number of different assertions. As there are 26 of these (art. 14, note,) there must be 26 × 26 = 676 possible syllogisms, each of which will give a different resultant. Of these 2 × 15 × 11 = 330 will have one of the 11 complex incomplete as one of their premisses, the other being complete or else simple incomplete; and 11 × 11 or 121 will have both premisses complex incomplete. These syllogisms have very little interest, but may all be worked out on the model of the third kind of syllogism, and as such form occasionally useful exercises. The remaining 225 may be distributed into 3 classes; 8 × 8 = 64 having both premisses incomplete, which are those considered in the text; 2 × 7 × 8 = 112 having one premiss incomplete and one premiss complete; and 7 × 7 = 49 having both premisses complete, which comprise all the subcases mentioned in the text under each kind of syllogism. To embrace all these 225 cases, 20 "skeleton rules" are necessary, which are here added, with the numbers of cases comprised in each, and one example in the short form, as they may be useful in suggesting exercises to teachers; each has been specially considered and worked in my *Contributions to Formal Logic.* Rules 1 to 5 belong to the first class, 6 to 13 to the second, and the remainder to the third.

1, 2, and 3 are the first, second, and third kind of syllogisms in the text. 8, 8, and 16 cases respectively. See arts. 22, 24, 26.

4. One premiss †, and one ‡. Means unlike. Conclusion, ?. Ex. †AE · ‡*e*I ∴ ? |AI|. 16 cases. See [4], art. 29.

5. Both premisses ‡. Means indifferent. Conclusion, ?. Ex. ‡AE · ‡EI ∴ ? |AI|, ‡AE · ‡*e*I ∴ ? |AI|. 16 cases. See [5], art. 29.

6. One premiss ‡†, and one †. Means unlike. Conclusion, ‡† extremes. Ex. ‡†AE · †*e*I ∴ ‡†AI. 16 cases.

7. One premiss ‡†, and one †. Means like. Conclusion, ‡ contraries of extremes. Ex. ‡†AE · † EI ∴ ‡*ai*. 16 cases.

8. One premiss ‡†, and one ‡. Means like. Conclusion, ‡ extreme of ‡ premiss with the contrary of extreme of ‡† premiss. Ex. ‡† AE · ‡ EI ∴ ‡*a*I. 16 cases.

9. One premiss ‡†, and one ‡. Means unlike. Conclusion, ?. Ex. ‡† AE · ‡*e*I ∴ ? |AI|. 16 cases.

10. One premiss ††, and one †. Means unlike, by arrangement of the

but this arose from insufficient knowledge. It may be convenient for the teacher to have the old forms of syllogisms translated into the present. They are therefore given here as Prof. Bain has presented them, with the old barbarous Latin names having their vowel quantities marked. The present letters A, E, I have been substituted for those of Prof. Bain. There are apparently 19 forms, but 8 are merely verbal varieties of other forms, depending on what is called *figure* and *mood*, (which again depended on the grammatical order of subject and predicate) distinctions rejected by De Morgan, and needless to be considered now. The bracketed numbers at the end refer to the three kinds of syllogisms and their rules, as given above, Arts. 23, 25, 27. Observe that by "all A is E," Prof. Bain means, in the present case, "all words containing A contain E," or †A*e*; by "some A is E," he means "at least one word containing A contains E," or ‡AE; by "some A is not E," he means, "at least one word containing A does not contain E," or ‡A*e*; and by "no A is E" he means "not even

†† premiss. Conclusion, † extremes. Ex. †† AE · †*e*I ∴ †AI. Use either †† AE or its equivalent ††*ae*, according to the phase of the mean in the other premiss, for the sake of the rule. 16 cases.

11. One premiss ††, and ‡. Means unlike, by arrangement of ††, as in No. 10. Conclusion, ‡ extremes. Ex. †† AE · ‡*e*I ∴ ‡ AI. 16 cases.

12. One premiss ‡‡, and one †. Means indifferent. Conclusion, ‡ the contrary of the extreme of the † premiss with the other extreme and also with its contrary. Ex. ‡‡ AE · † EI ∴ ‡(A*i*, *ai*). 8 cases.

13. One premiss ‡‡, and one ‡. Means indifferent. Conclusion, ?. Ex. ‡‡ AE · ‡ EI ∴ ? |AI|. 8 cases.

14. Both premisses ‡†. Means unlike. Conclusion, ‡† extremes. Ex. ‡† AE · ‡†*e*I ∴ ‡†AI. 8 cases.

15. Both premisses ‡†. Means like. Conclusion, ‡ the contraries of the extremes. Ex. ‡†AE·‡†EI ∴ ‡*ai*. 8 cases.

16. One premiss ††, and one ‡†. Means unlike, by arrangement of ††, as in No. 10. Conclusion, ‡† extremes. Ex. ††AE · ‡†*e*I ∴ ‡†AI. 16 cases.

17. Both premisses ††. Means unlike, by arrangement of ††, as in No. 10. Conclusion, †† extremes. Ex. ††AE · ††*e*I ∴ ††AI, which is the same as ††*ae* · ††E*i* ∴ ††*ai*. 4 cases.

18. One premiss ††, and one ‡‡. Means indifferent. Conclusion, ‡‡. Ex. ††AE · ‡‡EI ∴ ‡‡AI. 4 cases.

19. One premiss ‡‡, and one ‡†. Means indifferent. Conclusion, ‡ the contrary of the extreme of the ‡† premiss with the other extreme, and also with its contrary. Ex. ‡‡ AE · ‡†EI ∴ ‡ (A*i*, *ai*). 8 cases.

20. Both premisses ‡‡. Means indifferent. Conclusion, ?. Ex. ‡‡AE · ‡‡EI ∴ ? |AI|. 1 case.

It will be an excellent exercise first to form all the cases of each and apply the rule; secondly, to form the resultants of all, and show that even when the conclusions are the same the resultants are different; thirdly, to show that the conclusions in every case can be deduced from a continual application of the first five rules; and lastly, that every one of the 676 syllogisms furnishes a resultant and a conclusion consistent with one or more of the last 49, forming the third class, and for any determinate case of words selected must always be one or other of these 49. There is nothing in all this beyond the reach of intelligent boys that can do a sum in the rule of three.

one word contains both A and E," or †AE. Observe also that the *order* in which the premisses are arranged differs from that used in the preceding articles, but that this order is always indifferent, so far as the resultant and conclusion are concerned, though made a matter of some weight in the old logic. The mode of passing from the particular cases here worked, to the general case of ordinary logic, is given hereafter, Arts. 40 to 42. Examples of ordinary assertions will be found in Arts. 43 and 44.

FIRST FIGURE.

1. *Barbără.* All E is I, all A is E, ∴ all A is I; †E*i* · †A*e*, ∴ †A*i*, [1].
2. *Cĕlārent.* No E is I, all A is E, ∴ no A is I; †EI · †A*e*, ∴ †AI, [1].
3. *Dărĭī.* All E is I, some A is E, ∴ some A is I; †E*i* · ‡AE, ∴ ‡AI, [3].
4. *Fĕrĭō.* No E is I, some A is E, ∴ some A is not I; †EI · ‡AE, ∴ ‡A*i*, [3].

SECOND FIGURE.

5. *Cĕsărĕ.* No I is E, all A is E, ∴ no A is I; †EI · †A*e*, ∴ †AI. [1]. This is the same as No. 2.
6. *Cămestrēs.* All I is E, no A is E, ∴ no A is I; †*e*I · †AE, ∴ †AI, [1].
7. *Festīnŏ.* No I is E, some A is E, ∴ some A is not I; †EI · ‡AE, ∴ ‡A*i*, [3]. This is the same as No. 4.
8. *Bărōkŏ.* All I is E, some A is not E, ∴ some A is not I; †*e*I · ‡A*e*, ∴ ‡A*i*, [3].

THIRD FIGURE.

9. *Dărapti.* All E is I, all E is A, ∴ some A is I; †E*i* · †*a*E ∴ ‡AI, [2].
10. *Dĭsămĭs.* Some E is I, all E is A, ∴ some A is I; ‡EI · †*a*E, ∴ ‡AI, [3].
11. *Dătisĭ.* All E is I, some E is A, ∴ some A is I; †E*i* · ‡AE, ∴ ‡AI, [3]. This is the same as No. 3.
12. *Fĕlapton.* No E is I, all E is A, ∴ some A is not I; †EI · †*a*E, ∴ ‡A*i*, [2].
13. *Bŏkardō.* Some E is not I, all E is A, ∴ some A is not I; ‡E*i* · †*a*E, ∴ ‡A*i*, [3].
14. *Fĕrisŏn.* No E is I, some E is A, ∴ some A is not I; †EI · ‡AE, ∴ ‡A*i*, [3]. This is the same as No. 4.

FOURTH FIGURE.

15. *Brămantip.* All I is E, all E is A, ∴ some A is I; †*e*I · †*a*E, ∴ †*a*I, [1], which is the full conclusion, and this implies ‡AI (art. 14, vii), which is the old conclusion, where ‡AI is used instead of †*a*I, simply because the old logic had no terms to express the latter assertion as a conclusion, with A as the "subject." Sir W. Hamilton introduced the phrase: "some A is all I," for †*a*I, (art 43, vi. and note,) and dismissed the old form *Bramantip*.
16. *Cămĕnēs.* All I is E, no E is A, ∴ no A is I; †*e*I · †AE ∴ †AI, [1]. This is the same as No. 6.

17. *Dĭmăris.* Some I is E, all E is A, ∴ some A is I; ‡EI · †aE ∴ ‡AI, [3]. This is the same as No. 10.
18. *Fēsāpŏ.* No I is E, all E is A, ∴ some A is not I; †EI · †aE ∴ ‡Ai, [2]. This is the same as No. 12.
19. *Frĕsīson.* No I is E, some E is A, ∴ some A is not I: †EI · ‡AE, ∴ ‡Ai, [3]. This is the same as No. 4.

The teacher who has read Archbishop Thomson's Laws of Thought, will find 36 syllogisms given as Sir William Hamilton's, in each of the three figures, of which the Archbishop retains 22 in the first figure, and 20 in each of the others. All of Sir William Hamilton's, except 8, can be immediately solved by the processes here given; and of these 8, 4 belong to the class of doubtful conclusions, and the other 4 are really not syllogisms, because they implicitly contain a fourth name,* and, when solved, lead also to doubtful conclusions. None of them have been admitted by other logicians, and hence need not be further considered.

These are merely notes to help the teacher to translate his old books into these new symbols, and have nothing to do with the children. All remnants of merely medieval formalism and its consequences must be carefully kept from them. To resume:—

31. The three skeleton rules for syllogisms yielding certain conclusions, and the two rules for syllogisms yielding doubtful conclusions, will enable the pupil to solve an immense number of examples without hesitation. Suppose, for instance, we take the ten cases considered in the third kind of syllogism, art 26, (for example, the one ‡†A*e* · ††E*i*, ∴ ‡†A*i*,) we may solve each of them by a continual application of the rules. Thus ‡†A*e* means †A*e* · ‡*a*E, and ††E*i* means †E*i* · †*e*I. Taking each of the first with each of the second, we get cases to which the rules apply. Thus †A*e* · †E*i*, ∴ †A*i*, [1]; †A*e* · †*e*I, ∴ ‡*ai*, [2]; ‡*a*E · †E*i*, ∴ ‡*a*I, [3]; ‡*a*E · †*e*I, ∴ ?, [5]: the conclusion is then †A*i* (which implies ‡AI · ‡*ai*), ‡*ai*, (already implied in the last,) and ‡*a*I, that is, on the whole, ‡AI · †A*i* · ‡*a*I · ‡*ai*, or ‡†A*i* as already found. And so on for every possible case that can occur. These rules therefore embrace the whole ordinary theory of the syllogism, as enlarged by De Morgan and Hamilton.

* Thus one of these syllogisms, reduced to a case of letters in words, is: No words have E without I, at least one word containing A is different from at least one word containing E, ∴ at least one word containing A is different from at least one word containing I. Now the assertion that at least one word containing A is different from at least one word containing E, means that the first contains (or does not contain) some other letter, say N, which the second does not (or does) contain. Then we have ‡AN and ‡E*n*. There are therefore three premisses †E*i*, ‡AN, ‡E*n*. Then †E*i* · ‡E*n* ∴ ‡*n*I by [3], and ‡*n*I · ‡AN gives a doubtful conclusion, by [5], but contains the expression of Sir W. Hamilton's conclusion, for we see that at least one word contains I without N, and at least one word contains both A and N, so that these words must be different.

32. Again, the rules immediately enable the pupil to solve enthymemes; that is, when the conclusion and one premiss is given, the other can be found. Thus let the conclusion be †*a*I, and one premiss be †*a*E, then the other must be †*e*I, by [1], which shows that the conclusion †*a*I can only belong to the first kind. Hence also if ‡AE were given as a premiss, having †*a*I as a conclusion, we should see there must be an error. In fact, the only certain syllogisms with ‡AE as a premiss, are ‡AE · †EI, ∴ ‡A*i*, and ‡AE · †E*i* ∴ ‡AI, (art. 27.)

33. Again, given a conclusion and a mean, we can find all the syllogisms which will produce it. Thus: Let ‡AI be the conclusion, and E or *e* the mean; we see that †*a*E · †E*i*, †*ae* · †*ei*, †*a*E · ‡EI, †*ae* · ‡*e*I, ‡AE · †E*i*, ‡A*e* · †*ei*, and no others, will give ‡AI, the two first by [2] and the others by [3].

34. We can also see whether any given syllogism is logically bad. For, stating the premisses, we can immediately read off the correct conclusion, and compare it with the one given. Thus †AE · †E*i* ∴ ‡AI is wrong, the conclusion is ‡*a*I, leaving ?AI. Thus logical fallacies are detected. The greater number of fallacies in reasoning depend upon assuming incorrect premisses. One method of showing their error is to combine them logically with correct premisses and obtain conclusions properly drawn, which by their absurdity show the absurdity of the premiss which involved them.

35. We do not *disprove* a conclusion, by showing that either one or both of the premisses state precisely the contrary of what is true; for if we change ‡ and † into † and ‡ in either premiss separately or in both together, we get a conclusion which is consistent with the former. This should be shown for each kind of syllogism, thus—

First kind. †A*e* · †E*i* ∴ †A*i*, [1].
But †A*e* · ‡E*i* ∴ ?, [4]; ‡A*e* · †E*i* ∴ ?[4]; and ‡A*e* · ‡E*i* ∴ ? [5]; and a doubtful conclusion is consistent with any conclusion whatever.

Second kind. †*a*E · †E*i* ∴ ‡AI, [2].
But ‡*a*E · †E*i* ∴ ‡*a*I, [3]; †*a*E · ‡E*i* ∴ ‡A*i*, [3], and ‡*a*E · ‡E*i* ∴ ? [5]. Now ‡AI is consistent with either or both of ‡*a*I and ‡A*i*, see Art. 14, i.

Third kind. ‡AE · †E*i* ∴ ‡AI, [3].
But ‡AE · ‡E*i* ∴ ?, [5]; †AE · †E*i* ∴ ‡*a*I, [2]; and †AE · ‡E*i* ∴ ‡*ai*, [3]. And ‡AI is consistent with either or both of ‡*a*I, and ‡*ai*.

To disprove a conclusion, we must prove the correctness of an assertion which is inconsistent with it. Thus to disprove †A*i*, we must prove ‡A*i*, or †AI, or †*ai*, and the various ways of proving each of these with a given mean are obtained immediately from the skeleton rules, as shown in art. 33. But if none of the premisses required for this purpose are correct, we cannot disprove the conclusion, even if we do not admit the premisses on which it is based. Of course, it does not follow that the conclusion was right, but merely that we have not found out the proper form of mean for disproving it syllogistically; and the choice of means

is unlimited. More often, however, the conclusion is disproved by some of the inconsistent assertions being established by direct observation or experiment, and not syllogistically. The favourite method of disproving one or both of the premisses, merely shows that a wrong line of argument has been followed, not that the conclusion is wrong. Thus, assuming the transition from words and letters to ordinary instances, made in art. 40, let the argument be:—"*Pompey* (A_1) is a *man* (E); all men are *mortal* (I); hence Pompey is mortal." Here *Pompey* being an individual name is marked A_1, (art. 20,) so that the first premiss is ‡A_1E or †A_1e, and the second is †E*i*, the conclusion being †A_1i, by [1], giving ‡A_1I. Now suppose we deny †A_1e, and declare ‡A_1e, (Pompey is a dog, for example), and again deny †E*i* and declare ‡E*i*, (at least Enoch and Elijah were not mortal), then ‡A_1e · ‡E*i* ∴ ?, and does not show ‡A_1i (or that the dog Pompey is non-mortal!), that is, does not disprove †A_1i (or that Pompey, be he man or dog, is mortal).

36. On the other hand, if we take one of the premisses and combine it with an assertion which is inconsistent with the conclusion taken as a second premiss, we shall find as a conclusion something inconsistent with the second premiss. This is also best shown for each kind of syllogism separately.

First kind. †A*e* · †E*i* ∴ †A*i*, [1].

Assertions inconsistent with †A*e* are ‡A*e*, †AE, †*ae*
" " " †E*i* are ‡E*i*, †EI, †*ei*
" " " †A*i* are ‡A*i*, †AI, †*ai*.

Now combine †A*e* with each of the three last, and we shall get one of the three second in each case, thus:—

†A*e* · ‡A*i* ∴ ‡E*i*, [3]; †A*e* · †AI ∴ ‡E*i*, [2]; †A*e* · †*ai* ∴ †*ei*, [1].

Again combine †E*i* with each of the three last and we shall get one of the three first in each case, thus—

†E*i* · ‡A*i* ∴ ‡A*e*, [3]; †E*i* · †AI ∴ †AE, [1]; †E*i* · †*ai* ∴ ‡A*e*, [2].

Second kind. †*a*E · †E*i* ∴ ‡AI, [2]

Inconsistent with †*a*E are ‡*a*E, †AE, †*ae*
" " †E*i* ... ‡E*i*, †EI, †*ei*
" " ‡AI is †AI only.

Then †*a*E · †AI ∴ †EI, [1]; †E*i* · †AI ∴ †AE, [1].

Third kind. ‡AE · †E*i* ∴ ‡AI, [3]

Inconsistent with ‡AE is †AE only
" " †E*i* are ‡E*i*, †EI, †*ei*
" " ‡AI is †AI only.

Then ‡AE · †AI ∴ ‡E*i*, [3]; †E*i* · †AI ∴ †AE, [1].

These syllogisms are sometimes called the *opponents* of those from which they have been derived, but they have not been usually treated with proper fulness. For example, only two opponents have been allowed in each case, whereas it has just been shown that the first kind of syllogism has six opponents.

37. Will, then, one of the premisses combined with the conclusion as a second premiss, give the other premiss as a new conclusion? Try for each kind, taking the three examples just chosen.

First. †A*e* · †A*i* ∴ ‡EI, which is not †E*i*, but is consistent with it; and †E*i* · †A*i* ∴ ‡*ae* which is also consistent with †A*e*.

Second. †*a*E · ‡AI ∴ ? [4], and †E*i* · ‡AI ∴ ?, [4], which results are of course consistent with †E*i* and †*a*E respectively.

Third. ‡AE · ‡AI ∴ ?, [4], †E*i* · ‡AI ∴ ?, [4], which again are consistent with †E*i* and ‡AE respectively.

The new conclusion, therefore, in each case differs from, but is consistent with the other premiss.

38. We may therefore believe that if any three assertions consistent with the premisses and conclusion of a syllogism are assumed, and any two of them are taken as premisses, the conclusion will be consistent with the third. This belief will be raised to a certainty by considering that the premisses and conclusion are all three of them mere portions of the *resultant*, which was produced by the joint action of the premisses, so that if merely the resultant were given, all three assertions, namely the two premisses and the conclusion, could be deduced from it. This can readily be done by the process in art. 19, hitherto followed in obtaining the conclusion from the resultant. The Teacher is recommended to set up a resultant in counters, and obtain from it all the possible results respecting A,E and E,I as well as A,I.

The pupil may also be exercised in deducing other assertions respecting the three names A, E, I, from a given resultant. Thus, in the resultant in art. 22, all words containing both A and E also contain I. This is shown by ‡AEI and †AE*i*, which must necessarily involve the fact that at least one word which does not contain both A and E, that is, which is a non-AE-word, (and which is therefore an A*e*-, *a*E-, or *ae*-word,) does not contain I. Now the resultant agrees with this by means of ‡*aei*, although †A*e*I, †A*ei* show that no words have A without E, and ?*ae*I shows that although there is at least one word with neither A nor E (shown by ‡*aei*) yet, even if there are more such words, it is doubtful whether even one of them contains I.

Again ?*a*EI · †*a*E*i* show that though the existence of any words having E without A is doubtful, yet if any such exist, that is, if ‡*a*EI, they all will also contain I. The consequence of which is that there must *in that case* be at least one word which does not contain E without A, that is which is a non-*a*E-word, (and which will therefore be an AE-, A*e*-, or *ae*-word,) and also does not contain I; that is, that ‡^{1}(AE*i*, A*ei*, *aei*); and we see, in fact, that though †AE*i* and †A*ei*, yet ‡*aei*.

Judicious exercise of this kind, illustrated by examples in words and drawing of diagrams, will materially strengthen the reasoning powers, and at the same time may be made the source of much interest by presenting unexpected problems.

39. The method here adopted is complete and systematic, but the teacher may find it more interesting and instructive at first to adopt an unsystematic method, and then come to system as a means of classifying cases which have already occurred. The present principles with the use of counters are eminently adapted for such a course. Various cases of the same syllogism, arising from determining the ? compounds in the premisses as ‡ or †, have

also to be worked out and their results contrasted. This may be done without recourse to the skeleton rules, which may be reserved as convenient "skeleton keys" to pick future logical locks. It should be remembered that the real thing to be obtained and thoroughly understood is the *resultant*, which is altogether neglected in ordinary logical treatises, and that the conclusion is only the statement of *part* of the facts contained in the resultant, the *whole* of which can alone completely determine what is affirmed, denied, or left in doubt by the premisses, when taken jointly.

For these purposes the following modification of the forms in which the investigations were conducted will be found convenient. The table on this page shows how the ten definite forms of the third kind of syllogism may be exhibited at one view upon a slate or on the black board, and contrasted with the original indefinite form (involving doubtful compounds). The letters are first written down, and their columns for each case ruled. The letters for the premisses occupy the upper, and those for the resultant the

PREMISSES.

		1	2	3	4	5	6	7	8	9	10
‡	AE	‡	‡	‡	‡	‡	‡	‡	‡	‡	‡
2_1	A*e*	‡	†	‡	†	‡	‡	†	†	‡	‡
2_2	*a*E	†	‡	‡	‡	†	‡	†	†	‡	‡
$2_2\ 2_1$	*ae*	‡	‡	†	‡	‡	†	‡	‡	‡	‡
‡	EI	‡	‡	‡	‡	‡	‡	‡	‡	‡	‡
†	E*i*	†	†	†	†	†	†	†	†	†	†
?	*e*I	‡	‡	‡	†	†	†	‡	†	‡	†
‡	*ei*	‡	‡	‡	‡	‡	‡	‡	‡	‡	‡
‡AE †E*i*		‡†*a*E ‡†E*i*	‡†A*e* ‡†E*i*	‡†*ae* ‡†E*i*	‡†A*e* ††E*i*	‡†*a*E ††E*i*	‡†*ae* ††E*i*	††A*e* ‡†E*i*	††A*e* ††E*i*	‡‡AE ‡†E*i*	‡‡AE ††E*i*

RESULTANTS.

		1	2	3	4	5	6	7	8	9	10
‡	AEI	‡	‡	‡	‡	‡	‡	‡	‡	‡	‡
†	AE*i*	†	†	†	†	†	†	†	†	†	†
?	A*e*I	$2_6\ 2_4$	†	‡	†	†	†	†	†	$2_6\ 2_4$	†
2_3	A*ei*	$2_3\ 2_4$	†	‡	†	‡	‡	†	†	$2_3\ 2_4$	‡
3_1	*a*EI	†	‡	‡	‡	†	‡	†	†	‡	‡
†	*a*E*i*	†	†	†	†	†	†	†	†	†	†
3_1	*ae*I	$2_6\ 2_5$	‡	†	†	†	†	‡	†	$2_6\ 2_5$	†
$3_1\ 2_3$	*aei*	$2_3\ 2_5$	‡	†	‡	‡	†	‡	‡	$2_3\ 2_5$	‡
‡AI		‡AI	‡†A*i*	‡†*ai*	‡†A*i*	‡†*a*I	‡†*ai*	‡†A*i*	††A*i*	‡AI· ‡*a*I	‡‡AI

under part. The symbols ‡, †, ?, for the indefinite form, stand on the left; those for the various definite forms, on the right. Each premiss is written at full length. The several columns are numbered for convenience of reference. In the columns of the premisses appear those variations of the premisses enumerated at the close of art. 26, p. 27. The corresponding columns of the resultant are worked out from them in the way there explained, and the limitate indices are employed as there introduced. Under each column of the premisses are written the short forms of the premisses, and under each column of the resultant the short form of the conclusion. The order of the cases is that adopted in art. 26. This mode of working will be found convenient not merely for contrasting several definite forms corresponding to one indefinite form of syllogism, but for working out a number of unrelated syllogisms, without writing the letter part over and over again.

40. We are now prepared to make the transition from the particular case of words containing letters, to the general case of things possessing attributes distinguished by names, and thus to transfer all the knowledge and mechanical facility already gained to the propositions with which Logic ordinarily deals,—propositions which from their abstract nature are generally ill-adapted for elementary exercise.

The pupils will probably have been to a museum, or they may be taken to one, or to any labelled collection of objects, for the purpose of the present course.* Their attention must be drawn to the fact that each object is labelled, and that each label contains several names, and that these names have been given in order to recall to the mind of observers certain thoughts or feelings or sensations which they experienced when they fully examined the objects, not only externally but internally, not only by sight, but by touch, and occasionally the other senses, by breaking, dissecting, weighing, burning, trying with acids, observing habits of life, growth, and decay, if the object had been alive; in short, by every possible way that they could imagine which would lead them to distinguish one from another. That any simple name when placed upon an object implied a simple sensation experienced by the observer, but that, in general, names were put for shortness which implied a great many simple sensations, leaving it to dictionaries or special treatises to explain what they were. Perhaps the teacher may be able to go so far as to explain that there is presumed to be in each object an antecedent of such sensations, and that this antecedent is called an attribute, quality, or property of the object. This is a difficult matter to make clear, and it must be approached with caution, and always by drawing the thought from the pupil. Next make the pupil observe that these labels, and the words on them, bear a wonderful resemblance to words and the letters they contain; that if the labels were taken off you could sort them first

* Most schools have museums of quite sufficient extent for this purpose; but, if necessary, the furniture, books, ink-pots, slates, &c. in a school-room might be labelled for the occasion. Collections of shells, minerals, flowers, or stuffed animals are the most convenient.

into those containing such a name and those without it; then could sort the first set into those containing another name and those not containing it, and the second set likewise; and so on. We might, in fact, suppose that the names were represented by our letters A, E, I, &c., and that the labels not having the names A, E, I, had the letters *a, e, i* respectively, to show that the names A, E, I had not been omitted by oversight, but that the other objects to which these names had not been affixed really had not the names A, E, I. This once done, the labels and our previous slips of paper and counters become one and the same thing. But now the slips or labels no longer mean words; the capital letters A, E, I no longer indicate that the words contain those vowels, and the small letters *a, e, i*, that these words do not contain them. The slips or labels or counters now mean objects in the museum. The letters A, E, I are convenient abridgments of the names which the objects bore in the museum in order to certify that those objects had certain corresponding properties. The presence of the letters *a, e, i*, indicate the absence of the names denoted by A,E,I, and hence the absence of the corresponding properties in the objects. Also draw attention to the fact that many different objects have some one name on their labels, indicating the existence of the same property in all, and that almost all the objects have several different names on each of their labels, indicating that one object has several different properties. By this means the pupil will readily appreciate the facts indicated in art. 6.

41. Now make the transition to language in general. Show how we have named things, (taking material objects first, then their actions, &c.,) on account of some sensations they aroused in us, how we have applied the same name, as *man*, to various objects because many of the sensations they aroused were identical, and how, if each sensation had a different simple name, any name like *man* would be in fact a compound name made up of the names of all the different sensations which must concur in order for us to have the very compound sensation which the simple word *man* now recalls. Turn to a *bird*; elicit (rather than show) many properties in common between a man and a bird, as for example that both are animals and both two-legged. Then elicit their differences, as feathers, wings, &c., and lead to the conclusion that birds are non-men, and men are non-birds. Find other non-men, as quadrupeds, which are also non-birds. Now abbreviate the words Man into M, Bird into B, Two-legged into T, and Quadruped into Q, when of course *m, b, t, q* will be names of non-men, non-birds, non-two-legged, non-quadrupeds. Elicit the following expressions of observations made on these objects. Not only †M*t*, giving as a logical consequence, art. 14, vi., ‡MT and ‡*mt*, that is, "not even one man is non-two-legged, every man is two-legged, every thing which is not two-legged is not a man," but also, as a new observation,

‡*m*T, "at least one two-legged thing (as a bird) is not a man," so that the logical ?*m*T is determined, and we have really

‡†M*t* || ‡MT · †M*t* · ‡*m*T · *mt*, in which every compound must be thoroughly explained and exemplified by the pupils. Again, find

†MQ, giving as a logical consequence, art. 14, v., ‡M*q* and ‡*m*Q, that is, "*not one* man is a quadruped, every man is a non-quadruped, every quadruped is a non-man," and also, as a new observation,

‡*mq* "at least one non-man (as a bird) is a non-quadruped," so that the logical ?*mq* is determined, and we have really

‡†MQ || †MQ · ‡M*q* · ‡*m*Q · ‡*mq*, no term being left in doubt. Similarly deal with †B*t* and find ‡*b*T, so that really ‡†B*t*. Also find †BQ and ‡*bq*, so that ‡†BQ.

Next lead the pupil to observe that †M*t* and †B*t* show that whatever you can say of a Man or of a Bird, you really do say of a Two-legged thing, so that there is no property possessed by a man or a bird which is not possessed by at least one two-legged thing. (This is an important general principle, see p. 41, note *.) But as ‡*m*T and ‡*b*T, there is certainly at least one property possessed by at least one two-legged thing which is not possessed by any one man, and also at least one property, (*not* necessarily the same as the last,) which is not possessed by any one bird. Show that if it were otherwise, that is, if we had †*m*T and †*b*T, in addition to †M*t* and †B*t* we should have ††M*t*, and ††B*t*, in which case every single two-legged thing would be both a man and a bird!!

42. Thus the assertions which we have lately considered respecting the existence of certain letters in certain words, can all be read as assertions respecting the attributes residing in certain things. In fact all our thoughts may be considered as slips of paper or counters, each containing some one distinctive mark or name which "denotes" the particular slip or counter, and each containing the marks or names of the particular attributes of those thoughts which are "con-noted" by that first distinctive mark. Once reduced to this form, all assertions whatever become precisely the same as assertions respecting letters in words, where the *word* forms the "de-notation," and the several letters, being the whole of the distinctive marks or attributes in the word, are its "con-notation."*

43. A large number of assertions must now be analyzed. Examples may be collected from all elementary treatises on Logic.

* The *order* of the letters is important in a word, and must be considered as an attribute. But this may be here disregarded. Thus, so far as possessing the letters I,P,T, and no others, *pit* and *tip* are the same. But in writing down the letters we introduce a new attribute, order. In thinking of things, this order is unimportant. Whether we say a bird is two-legged and feathered, or feathered and two-legged, is of no consequence. This little point of difference should be borne in mind by the teacher, but need not be mentioned unless he sees that a difficulty arises in the pupil's mind; and then the explanation must be elicited by questioning. The distinction, however, between the two cases is that mathematically important distinction between commutative and non-commutative operations; and Boole's mathematical theory of logic depends mainly upon the commutative character of attributes, or indifference in the order of mentioning them.

The following specimens will show the relations of the present strict notation to common language, and some of the difficulties to be contended with.* For convenience some of the words are placed in ( ), and others in [ ]; the letters A and E refer to these words respectively, so that the expressions have the same form as those considered in arts. 13 and 14. But in practice the letters should be varied, and may generally be chosen so as to recall the words, as in art. 41.

i. All (the righteous) are [happy], †A*e*, or "not one righteous man is non-happy;" implying ‡AE, or "at least one righteous man is happy," and ‡*ae*, "at least one non-righteous man is non-happy," and leaving absolutely doubtful, whether there is or is not even one non-righteous man who is happy, or ?*a*E.

ii. No (human virtues) are [perfect], †AE, implying ‡A*e* and ‡*a*E, but leaving ?*ae*. In putting these into language, remember *a* is the name of a non-human, not of an in-human, virtue. The forms †AE·‡*a*E show that all perfect virtues are non-human. If we were to write "all perfect virtues are not human," the phrase would be ambiguous, and might also mean either ‡AE, ("although *all* perfect virtues may not be human, *some* are"), which is of course wrong.

iii. Some (possible cases) are [probable], ‡AE, or "at least one possible case is a probable case." Observe that "some" in Logic

* The first six of these specimens are the six "judgments" of Archbishop Thomson ("Laws of Thought, § 78, p. 135 of tenth edition, 1869), which he symbolizes, in order, by the capital letters A, E, I, O, U, Y. In Sir W. Hamilton's formal language (*ib.*, § 79), translated into the present symbols, they are—

i. A.	All A is some E	†A*e*	iv. O.	Some A is no E	‡A*e*
ii. E.	No A is E	†AE	v. U.	All A is all E	††A*e*
iii. I.	Some A is some E	‡AE	vi. Y.	Some A is all E	†*a*E

Archbishop Thomson calls these "*all* the judgments," as he does not admit the two following of Sir W. Hamilton, which, however, he cites and symbolizes by the Greek letters η, ω (*ib.*) :—

vii. η. No A is some E ‖ ‡*a*E | viii. ω. Some A is not some E ‖ ‡(AN, E*n*).

Another form of this last assertion is somewhat enigmatically stated by Sir W. Hamilton as: "some A is not some A." This reads like a contradiction in terms, but really means ‡(AN, A*n*), which is the same as art. 14, note, i. 1) The assertion ‡(AN, E*n*) states that there is at least one thing called A which (possessing the attribute N) is different from *at least one* thing called E (which does not possess the attribute N). 2) The assertion ‡(AN, A*n*) states that there is at least one thing called A which (possessing the attribute N) is different from *at least one* (other) thing called A (which does not possess the attribute N); that is, that there are at least two things called A (one with and one without the attribute N). 3) The assertion ‡A*e* shows that there is at least one thing called A which (not possessing the attribute E) is different from *every one* of the things called E. 4) The assertion ‡*a*E shows that there is at least one thing called E which (not possessing the attribute A) is different from *every one* of the things called A. 5) And the assertion ‡(A*e*, *a*E), art. 14, note, vi., makes both the third and fourth of these assertions at once, and consequently implies the first; for even if there be only one thing called A (and not

means "at least one," or "at least a portion of," and though it cannot be *none*, it may be *all*. Now this is different from its use in common life. Hence "at least one," is preferable. To say "at least one possible case is non-improbable," is merely putting *non-improbable* for *probable*, which will be correct if *improbable* only means *non-probable*, but not otherwise.

iv. Some (possible cases) are not [probable]. ‡A*e*, or "at least one possible case is a non-probable case." This is the method in which such sentences have to be understood.

v. (The just) are [the holy], ††A*e*, that is the same set of people is called both *just* and *holy*, not one who is just is non-holy, not one who is holy is non-just. Observe the extreme brevity of common language and the consequent probability of its being misunderstood. It implies ‡AE·‡*ae*; and consequently that "all the just are holy, all the holy just, all the non-just non-holy, and all the non-holy non-just." Observe, however, that though in this case the just *persons* are "identical" with the holy *persons*, within the range of thought, the property of being *just* is quite different from the property of being *holy*, and that, in fact, the use of the assertion is not to declare the identity in the *meaning* of the words, but only in the things to which they apply. (See art. 13, vii.) *This* "identity" must be kept quite distinct from *mathematical* "identity."* The pupil must be taught due caution in the use of such extremely hazardous words as "identical" and "same."

possessing the attribute E), and only one thing called E (and not possessing the attribute A), these two things must, in this case, be different. The difficulty usually experienced in understanding the assertions η, ω will justify the insertion of this explanation for the use of teachers. Their awkward wording by Sir W. Hamilton depends on his "quantification of the predicate," which is further referred to in art. 44, note, p. 45. The whole of this imperfect enumeration of assertions is superseded by the complete enumeration of assertions respecting two elements in arts. 13 and 14, including the note. The assertion ω in the form ‡(AN, A*n*) belongs to these, as we have seen. In the form ‡(AN, E*n*), it in fact consists of two premisses of a syllogism, as in art. 30, first note, No. 5, p. 29, which gives the resultant—

$$2_1\,\mathrm{AEN},\quad 4_4 2_1\,\mathrm{A}e\mathrm{N},\quad 4_3\ \ a\mathrm{EN},\quad 4_4 4_3\,ae\mathrm{N},$$
$$2_2\,\mathrm{AE}n,\quad 4_4\ \ \mathrm{A}en,\quad 4_3 2_2\,a\mathrm{E}n,\quad 4_4 4_3\,aen,$$

where the limitates 2_1 and 2_2 are due to ‡AN and ‡E*n* respectively [conditioning ‡(A, E, N, *n*)], and the limitates 4_3 and 4_4 to ‡*a* and ‡*e* respectively. Hence it is erroneously placed among assertions respecting two elements, and consequently, when used as a premiss, leads to pseudo-syllogisms, as explained in the second note to art. 30, p. 32. This analysis of Sir W. Hamilton's assertion ω will show how the method here proposed for teaching children the most elementary notions of logic, is in fact a powerful instrument for examining the laws of thought propounded by some of our profoundest thinkers.

* Two mathematical operations of a distinctly different form are said to be identical when they possess a common property—namely, that of both producing a given result according to the mere laws of operation, without any annexed condition. Two identical mathematical operations are consequently merely members of the same class (art. 44, i.), which may, and

vi. Some (happy persons) are all [the righteous]. †*a*E.

vii. All (the insincere) are [dishonest]. †A*e*.

viii. No (unjust act) is [unpunished]. †AE.

ix. Some (unfair acts) are [unknown]. ‡AE.

x. Some (improbable cases) are not [impossible]. ‡A*e*.

xi. The (unlawful) is the only [inexpedient]. ††A*e*.

xii. Some (unhappy men) are all the [unrighteous]. †*a*E.

xiii. (Fixed stars) are [luminous]. †A*e*.

xiv. (Life) every [man] (holds dear). †A*e*.

xv. Few, not a few, many, a large number of, the majority of, the minority of, most, not all, only some, not nearly all, some, at least two or three, one or two, (men) are [honest]. ‡AE. In ordinary logic, none of these quantitative words can be distinguished. They all agree in this, that "at least one A-thing is an E-thing." It might be at first supposed that they also agreed in this, that "at least one A-thing is an *e*-thing," or ‡A*e*. Persons who use the words may often mean both ‡AE and ‡A*e*, but on examination it will be found that the actual assertion is only ‡AE, and that the rest is left in doubt. The consideration of a definite number of things, or a definite part of the whole number of the things mentioned, belongs to a peculiar branch of logic, first treated by Prof. De Morgan, with which children must not be troubled, see art. 48. Observe that "not all (men) are [honest]" is ambiguous, and might also mean, "no (man) is [honest]," or †AE.

xvi. Every (mistake) is not [culpable]. This is ambiguous; either "not one (mistake) is [culpable]," †AE, or "at least one (mistake) is non-[culpable]," ‡A*e*.

xvii. None but the (brave) [deserve the fair]. †*a*E. The statement is apparently entirely negative. Show that it is equivalent to "all [men who deserve the fair] are (brave)," leaving ?A*e*, or a doubt as to whether those who do not deserve the fair include even one brave man.

xviii. Nothing is (beautiful) but [truth]. †A*e*.

xix. All (planets) are not [self-luminous]. Ambiguous, either "not one planet is self-luminous," †AE, or "at least one planet is self-luminous", ‡AE, which would agree with ordinary language.

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generally does, include many others. But members of the same class may always be substituted one for another, as long as merely their class property is concerned (see the general principle in art. 41, p. 39). The whole of algebraical reasoning consists, ultimately, in repeated substitutions of this kind. Even when a condition is annexed, it is supposed to be fulfilled for the purpose of carrying out the substitution, and it is itself generally determined by that means. Thus, $5+6x$ will be mathematically identical with 23, if x represents 3; and this condition is at first supposed to be unknown and yet fulfilled. Its exact nature is then determined by a succession of substitutions, as in the usual solution of simple equations, thus: $5+6x=23$, $(5+6x)-5=23-5$ or $6x=18$, whence $6x\div 6=18\div 6$ or $x=3$.

xx. (He) is [no fool]. †A_1*e*. Observe "he" is *singular*, and hence A_1 is used, see art. 20. Also observe that *no-fool* is much more than *a-non-fool*, or *not a fool*, and really implies the possession of a considerable amount of wisdom, or at least cleverness.

44. As examples of the great diversity of expression in use for the same assertion, take the following, which will guide the teacher in checking interpretations given by the pupil, who must be made to see in time, by a variety of concrete examples, and first by means of words containing letters, that each assertion has precisely the one same definite meaning here assigned to it, and more fully explained above as the fifth and second usual assertions, art. 14.

i. †P*q* implying ‡PQ · †P*q* · ?*p*Q · ‡*pq*.

Not even one thing (or object of thought) exists within the range of thought bearing the name P, (or possessing the attribute P), without also bearing the name Q (or possessing the attribute Q).

All P is Q. All Ps are Qs.

All P is some Q. Every one of the Ps is some one of the Qs.

Every P-thing is a Q-thing.

No P-thing is any non-Q-thing.

Everything is either Q or non-P, that is, either a Q-thing, or a non-P-thing.

Whatever is not called Q, is not called P.

Nothing which is not Q, is P.

Every non-Q-thing is a non-P-thing.

All non-Q is non-P. All non-Q is some non-P.

The class Q contains the class P, or the class P is in the class Q.

The class P has the name Q as one of its class-marks.

The attribute Q belongs to the things in the class P.

The name Q is an essential* part of the name P.

The attribute Q is an essential part of the attribute P.

The names of the things P are to be sought only among the names of the things Q.

The attribute P is only to be found among things possessing the attribute Q.

The things P are a species of the things Q.

The things Q are a genus of the things P.

The name P is dependent on the name Q.

All the properties of the things Q are to be found among the properties of the things P.

ii. ‡PQ implying ‡PQ · ‡[1](P*q*, *pq*) · ‡[1](*p*Q, *pq*.)

At least one thing (or object of thought) exists within the range of thought bearing both the names P and Q, (or possessing both the attributes P and Q).

* The confusion attending the use of the word "essence," is and will probably remain enormous. But, at least in early teaching, the old philosophical questions must not be raked up, and we must be content with such a statement as this, which, if not the *whole* truth, in the opinion of many thinkers, is at least all that comes into action in logic, the rest belonging to investigations into the "nature of things."

Some P is Q. Some P is some Q. Some Ps are some Qs.
Some Q is P. Some Q is some P. Some Qs are some Ps.
Some one of the things called P is also some one of the things called Q.
Some P is not any of the non-Qs.
The class non-Q does not contain the whole of the class P.
The class non-P does not contain the whole of the class Q.
All of the class P is not in the class non-Q.
At least one of the class P is in the class Q.
At least one of the class Q is in the class P.
The classes P and Q have at least one thing in common.
The class P may contain the class Q, or the class Q may contain the class P.
The absence of the name Q is not essential to the name P.
The absence of the attribute Q is an inessential of the attribute P.
The things P are not to be sought for only among things which have not the name Q.
The attribute P does not exist only outside of the class Q.
The things P are not a species of non-Q.
The things non-Q are not a genus of P.
The name P is independent of the absence of the name Q.
All the properties of the things non-Q are not to be found among the properties of the things P.

The same sort of phrases may be applied in case of the other assertions, as †PQ, ‡P*q*, &c., and the older pupils, at least, may be exercised in altering the terms accordingly. These modes of expressing assertions may be made the means of explaining such terms as class, genus, species, essential, &c., but are not suited for very young pupils. At this period also the use of the terms subject and predicate may be explained, if desired; but they have been the source of so much confusion, that they had better be left for a more advanced stage.*

* The real intention of the grammatical forms, from which the logical terms subject, copula, and predicate, have been deduced, was to express the relation †A_1e, in its most common form ‡†A_1e, which implied that the one thing called A_1 was also called E, and that other things not called A_1 were also called E, in English "A_1 is E." In short, A_1, the subject, was an individual in the set of things E, the predicate, and "is" the copula (which in many languages is only expressed by the relative position of the words) was merely a means of showing that the two words related to the same thing. But this ‡†A_1e, thus expressed, showed that the one thing A_1, and others also, for example, B_1, C_1, D_1, possessed the attribute E. Then, instead of speaking of A_1, B_1, C_1, D_1 separately, they were spoken of as "the E things," and when it was wished to say that A_1, B_1, C_1, D_1, and other things possessed the attribute F, the class E was made a subject and the class F a predicate in the form "E is F" or ‡†E*f*. Proceeding further in knowledge, A_1 and B_1 were found to possess the attribute G, but C_1 and D_1 had not that attribute. Here selections of the E class had to be made, and each considered as different subjects. This was done by means of the word *some*, or its equivalent in other languages, and "some E" became a *quantified* subject, as in "some E is G," or ‡EG. Then for a distinction (which, however, is generally omitted in ordinary

45. Definitions all depend upon the complete assertion ††P*q*, showing that the two names P and Q apply to the same things and no others, (Art. 13, No. vii). But care has to be taken that definition and connotation are not confused. Supposing that all P-things are M-things, or †P*m*, and that all P-things are also N-things, or †P*n*, and also that all things which are at once M-things and N-things are also P-things, or †*p*MN, then it is clear that if we have once ascertained that a thing has both the attributes M and N, we have found that it is P, and if we find that it has the attribute P we shall know that it has both the attributes M and N. But P need not *mean* MN; that is, MN need not be the connotation of P. Thus in a certain room we may know that all Englishmen wear blue coats and buff waistcoats. This becomes

language), the first relation ‡†E*f* had also to be *quantified*, as "all E is F," and, as now the ‡*e*F was immaterial, this phrase came to mean (in logic at least, though ordinary speech is ambiguous) only †E*f*, and not necessarily ‡†E*f*. But there was still a difficulty. How were we to declare an assertion to be false? It was clear that to say "all E-things are F-things," or †E*f*, and also "some individual E-things are non-F-things," or ‡E*f*, would be inconsistent. In the same way, "some E-things are G-things," or ‡EG, and "all E-things are non-G-things," or †EG, would also be inconsistent. But it did not occur to makers of language to talk of non-F-things, or non-G-things. Their corresponding terms are not so precise, and often go beyond the mark. Hence the difficulty was got over in one case by altering the copula, "some E *is not* G," or ‡E*g*, and in the other by *nullifying* the subject, "no E is G," or †EG. But great confusion then arose about ‡EG and †EG, for it was soon found that we might read them the other way over, and say with equal truth, "some G is E," or "no G is E," thus making the predicate into a subject, which was a complete perversion of the original notion. Hence arose all the logical troubles respecting *conversion*. The forms ‡EG, †EG, could be converted, but the forms ‡E*g*, †E*g*, could *not*. Why? We might suppose that it was easy to say, "some non-G-things are E-things," and "no non-G-things are E-things." But it was not, to minds which had not conceived of non-G-things as an existent reality, but only as a negation of G-things. This difficulty clung to all logic down to the time of Prof. De Morgan. It is however probable that conversion, by which the predicate became a subject, joined to the old quantification of the subject, led Sir William Hamilton to that unfortunate conception of the "quantification of the predicate," and that really barbarous terminology and abuse of language, into which one could scarcely have believed it possible for such a man to have floundered. It is useless to pursue it here. The hopeless confusion already created by the attempt to bend popular terms into scientific instruments, has been the main difficulty in logical treatises. The old imperfect usages of the first employers of language, which gave the distinction of subject and predicate, with the attempt to carry it through, misled such men as Aristotle into the imperfect theory of *inclusion* and *exclusion* (*dictum de omni et nullo*) as the foundation of logical thought. All this is entirely overcome by the notation here explained, and it would be confusing to a child's mind to perplex it with the meaning of such terms as subject, copula, and predicate, except as grammatical notions. In time the grammaticised pupil will be sure to ask the teacher of logic about it; and for that eventuality this note has been written, to show the teacher the relations which are really meant to be expressed.

for the moment our *definition* of an Englishman, the marks by which we find him out. But it is not the connotation or meaning of the word Englishman. Bearing this in mind, the teacher will have no difficulty with the cases treated in other books, and with the distinctions of *essence* (which belongs to the connotation) and *accident* (which does not belong to, but is not inconsistent with, the connotation). Observe that in the last case †P*m*, †P*n*, and †*p*MN, gives, by the general condition, ‡PMN · †(PM*n*, P*m*N, P*mn*, *p*MN) · ‡^{1}(*p*M*n*, *pm*N, *pmn*) · ‡^{1}(*pm*N, *pmn*) · ‡^{1}(*p*M*n*, *pmn*); that is, in the instance given, there will be in the room at least one Englishman with a blue coat and buff waistcoat, and at least some one who is not an Englishman, that will either not have a blue coat, or not a buff waistcoat, or have neither one nor the other; and there will be also at least one person without a blue coat, who is certainly not an Englishman, but may or may not have a buff waistcoat; and lastly, there will be at least one person without a buff waistcoat, who is certainly not an Englishman, but may or may not have a blue coat.

46. Disjunctive assertions can now be understood. Thus "all A is either B or C," means simply †A*bc*, or the name A is never found without at least one or other of the names B and C. But the assertion is ambiguously worded, for it may or may not imply that A can be both B and C. If A cannot be both, we have †ABC in addition. Hence the assertion "all A is either B or C" means, at full, *either* †A*bc* · ‡^{1}(ABC, AB*c*, A*b*C), *or* †(ABC, A*bc*) · ‡^{1}(AB*c*, A*b*C), the remaining compounds being doubtful. Whenever an ambiguous assertion is made, *both* its meanings should be discovered, and in all trains of reasoning, each of the meanings should be separately considered, and then the various conclusions should be separately exhibited, as if each had been originally derived from a single unambiguous assertion. This is the only certain way to avoid error. Other ambiguities are pointed out in art. 43. All such ambiguities should be avoided, if possible, and the use of the present symbols in place of or as an aid to ordinary language, will render their avoidance always possible. When the second kind of assertions are dealt with, similar ambiguities will also arise, as will be seen later on, (art. 59, i, j. 66, 67, 68).

47. The impossibility of referring to any text-book to explain the method of teaching logic to children, which I am here endeavouring to indicate, must be my apology for entering at so much length into minute details. What has been here advanced will however suffice, I hope, to enable any teacher who has himself studied some elementary treatise on logic,* to apply the

* *W. Stanley Jevons*, Elementary Lessons in Logic, deductive and inductive, with copious questions and examples, and a vocabulary of logical terms, London, Macmillan and Co., 1870, small 8vo., pp. 340, contains almost all that is wanted. Lesson 23 contains Prof. Jevons's own arrangement of Dr. Boole's system, which is the foundation of that here adopted, and it is my duty to mention that it was through Prof. Jevons's Substitution of Similars, 1869, and Pure Logic, 1864, that I was induced to study

method I advocate to every case which presents itself, and to solve every example furnished so far forth as the simple assertion and syllogism are concerned.

48. But the simple assertion and the syllogism are the mere beginnings of logical studies. Far more complicated assertions involving the composition of any number of names, and combined in any variety of ways, have to be considered by the advanced student, but of course cannot be presented to a child, as much more highly developed mental powers, combined with a much greater range of general knowledge, are required to comprehend the mere intention of the investigations. Much might be done by extending the examples of words containing or not containing certain letters; but the more advanced student would feel this to be a childish game (as in fact it is, and is meant to be,) unless he could feel that it had a real application to the nature which surrounds him. Hence I do not recommend proceeding beyond the point already reached, with assertions of the nature hitherto considered. Numerically definite assertions, although extremely important, and really easily exemplified by definite collections of words containing or not containing certain letters, are also scarcely adapted for children, even when presented in the simple manner here advocated. The utmost that can be done in this way, is to make clear the very simple case, that if *most* of the words in a given collection contain A, and also *most* of them contain E, then *at least one* of them contains both A and E. This is easily illustrated by a case like this.

A A A A A A *a a a a a*
e e e e e E E E E E E.

Dr. Boole's abstruse mathematical system of logic. As my plan bears in its foundations a great resemblance to Prof. Jevons's, they might appear identical on a superficial examination. It is necessary therefore to state, that the resemblance is entirely superficial, and that my views are not only more extensive than Prof. Jevons's, but aim at accomplishing all that is contained in *Dr. George Boole's* Investigation of the Laws of Thought, on which are founded the mathematical theories of Logic and Probabilities, (London, 1854, 8vo., pp. 424,) without his mathematical expression, and without those doubtful points of theory on which his mathematical views are based, and also at including the whole of *Prof. Augustus De Morgan's* Formal Logic, or the Calculus of Inference, Necessary and Probable, (London, 1847, 8vo., pp. 336), and his more recent Syllabus of a proposed System of Logic (London, 1860, 8vo., pp. 72), and of furnishing a complete explanation of the fundamental theories of Sir William Hamilton's New Analytic, as presented in *Archbishop William Thomson's* Outline of the necessary Laws of Thought, a treatise on pure and applied Logic, (London, 10th thousand, 1869, 8vo., pp. 304). Some account of these three latter systems is given in *Prof. Alexander Bain's* Logic, Part I., Deduction, (London, 1870, 8vo., pp. 279), to which, and to *Mr. Thomas Fowler's* Elements of Deductive Logic (3rd ed., Oxford, 1869, pp. 176), the teacher who shrinks from studying Archbishop Thomson, or the great works of Dr. Boole and Prof. De Morgan, is referred for further information. *Dr. F. Ueberweg's* System of Logic and History of Logical Doctrines, translated by T. M. Lindsay, (London, Longman, 1871, pp. 590, 8vo,) gives a full historical account of the old logic.

Here are 12 words, of which only the A's and E's are written, of which 7 contain A, 7 contain E, 5 do not contain A, and 5 do not contain E. By arranging them as above, it is clear that at least 2 must contain both A and E; but as they could be arranged thus—

A A A A A A A *a a a a a*
E E E E E E E *e e e e e*,

we might have as many as 7 words containing both A and E, and may have any intermediate number between 2 and 7. Particular cases of this simple nature may be readily solved by elementary arithmetic: but the general statement involves algebraical considerations.

Passing over these, and assertions respecting pure combinations, we come to what really form the most frequent and most important case,—assertions respecting the consistency of other assertions. The greater part of all actual ratiocination turns upon the test of consistency, and the whole doctrine of probabilities can be made to rest upon its proper development. Yet in ordinary treatises on Logic this principal part of the subject is usually dismissed in a few pages, with one or two almost self-evident examples, under the head of Complex or Hypothetical Propositions and Syllogisms. At the end of a lecture already far too long, it would be out of place for me to attempt giving this class of assertions their proper position; and, fortunately for my present purpose, the real development of their theory is so far beyond any child's mental powers, that it would be useless to make the attempt in these hints for teaching logic to children. But I must endeavour to give some account of the elementary notions, not only because they are usually insufficiently explained, but because some of the most ordinary processes of reasoning in common life, and in treatises on geometry, turn upon their application.

49. We are already familiar with cases of consistent and inconsistent assertions (art. 15). In the syllogism we have learned that the premisses and conclusion are all consistent with the resultant and with one another, and that although assertions inconsistent with the premisses are not necessarily inconsistent with the conclusion, they will necessarily be inconsistent with the resultant (arts. 31 to 39). In this consideration of consistency, we did not trouble ourselves with the *truth* or *correctness*, that is, the consistency with actual fact, of the assertions themselves. But it will be more convenient in future to speak of this *truth*, not as of any importance in itself, but as being consistent or not with the truth of other assertions. "*Supposing* an assertion to be true, will other assertions be true or false?" This is the question which lies at the root of all investigations into the consistency of assertions, and from this usual method of proposing it came the denomination "hypothetical." Again, the results are frequently stated in the form, "either this case or that will be true," and hence the name *disjunctive*, which implies that the two cases are inconsistent and alternative. Another form in which this case presents itself is: "Whenever or wherever this happens, that will happen." This is a purer form of stating the question of consistency. What-

ever form however be taken, *consistency* is at the root of the investigation. It is enough to state this, in order to show its radical importance to all thinkers, especially the least advanced, who are most exposed to the besetting sin of all speakers, (not to say arguers,) *consistent inconsistency*.

50. Now we cannot advance far without a notation less cumbrous than that of ordinary language. Letters must be employed, which must, as before, be considered as labels, or titles used in place of the whole expression of the matters, sometimes very lengthy, with which we have to deal. Let any capital letter, as X, signify the *truth* of a certain *assertion*, which for similarity may be written X′, (read X-*dash* or *assertion*-X), concerning a certain *event*, which, also for similarity, may be written X″ (read X-*double-dash* or *event* X).* The pupil will therefore learn to associate the conception of *truth*, *assertion*, and *event*, with *unaccented*, *once* accented, and *twice* accented letters, such as X, X′, X″. It is totally indifferent what the assertion, or event may be. But when several assertions are made respecting different events, different letters must be employed, distinguished by the same accents. Thus, if the assertion be "a house has fallen," and we call this assertion X′, then the event of the fall of the house will be X″, and the truth that the house has really fallen will be X simply. This will be a difficult thing for a child to understand, and great care must be taken not to hurry over this step. Pupils must themselves make and note different assertions in this way, till they become familiar with these *abbreviations*, for they are nothing more.

51. Now, if the assertion X′ is not true, if the event X″ has not happened or will not happen, there will be some other assertion which is true, some other event which does happen, and this assertion and this event will have been incompatible or inconsistent with X′, X″. Call this assertion *x*′, and this event *x*″, and the truth of *x*′ call *x*, reading *x* as non-X. The truths of X′ and *x*′ are inconsistent; that is, X and *x* are inconsistent. We *cannot* have *both*, X and *x*, but we *must* have *one* or the other. The assertion X′ must be true or false. If it is true, we have X; if it is false, we have *x*. This must also be well felt, be quite familiar, before the pupil advances at all. Draw his attention to the fact that *x*′ may represent *any one* of several assertions. Thus, if X′ be †AE, any

* In place of X′, X″, I have used αX, εX, in my *Contributions to Formal Logic*. These forms might be used, if preferred, the Greek α, ε being only considered as peculiar ways of writing *a*, *e*. Otherwise *as.* X, *ev.* X, or even *assertion* X, *event* X, might be used. The teacher must suit himself to the powers of his class. In my *Contributions*, &c., I also use τA for the *attribute* A, and θA for the *thing* possessing the *attribute* A, and hence called by the *name* A. The use of νA for *the number of the things* θA under consideration, and νX for *the number of times that the event* εX *occurs*, is also convenient in the investigation of numerically definite assertions and statistical conditions of probabilities; but these matters require a knowledge of algebra, and lie far beyond anything here attempted.

one of the assertions ‡AE, †Ae, †aE will be inconsistent with it, and hence any one of these will be represented by x', because the truth of any one of these implies the falsehood of X′. Hence although X′ may be single, x' may be ambiguous. Truth we know is one. error hydra-headed. If X″ does not happen, any one of several events x'' may have prevented it. "There is *many* a slip 'twixt the cup and the lip." But this is of no matter; we are only interested in knowing that if X″ occurs, x'' does not occur, and if x'' occurs, X″ does not occur.

52. This *occurrence* and *non-occurrence* of X″ or x'' may be considered as the *presence* or *absence* of X and x, and hence we may use our old signs of black and red letters on counters, and, in writing, our old signs ‡ and †. We do not require a doubtful sign, because *one or other must occur*, there is no doubt about it. Then ‡X (or black side of the counter X) indicates the *presence of the truth* of the assertion X′ concerning the event X″, and consequently implies †x (shown by the red side of counter x) or the *absence of the truth* of the assertion x' concerning the event x''; one of the assertions X′, x' must and only one can be true, that is, one of the two events X″, x'', must and only one can happen. In writing, we use $‡_1$ (X, x) to indicate this, where $‡_1$ read "present only one of," is the sign introduced in art. 20. With counters it is sufficient to place them on their sides, thus $\boxed{X \quad x}$. All counters so placed will be subject to this law, or form a group, called a *unicate*, of which one must be present, and only one can be present, although the conditions assigned have not been sufficient to determine which one that will be.

53. If two events X″, Y″ are *compatible*, or *compossible*, the two assertions X′, Y′ are *consistent*, and hence the two truths X, Y may *co-exist*. The two events may then be said to form a *complex* event, (X.Y)′, [where the full stop or dot (.) is read "*cum*,"] the two assertions form a complex asertion (X.Y)′; and the two truths form a complex truth, or simply a *complex* X.Y. Here be careful to distinguish a *complex* X.Y, (two truths, X and Y, considered as coexistent), from a *compound* AE (the name of a thing to which each of the names A and E apply singly), and draw attention to the presence and absence of the full stop or dot (.), as characterising the two symbols, X.Y and AE. Also distinguish this dot on the line (.), used for forming complexes, from the dot above the line (˙), used for connecting assertions. (See art. 12, on p. 13, l. 6.) Observe that the expression X.Y (the counter on its side) only states that X and Y *may* coexist. That they *do* exist is expressed by ‡X.Y (the counters erect, black side to the front), read "present X *cum* Y"; and that they do *not* coexist is expressed by †X.Y (the counters erect, red side to the front), read "absent X *cum* Y." The nature of the truths X and x is then such that †X.x, at all times and under all circumstances. This may be compared with †Aa, which expresses the fact, stated on p. 8, l. 5 from bottom. The difference between †X.x and †Aa must be thoroughly seized.

54. Observe that with X, there must always be either Y or y. No event occurs by itself. When it does occur, it is always accompanied by some other event, which is compatible with it, and that other event is again either compatible or not compatible with some other event. Hence *if* X″ occurs, either Y″ or y'' must occur, but both cannot occur. Hence if ‡X, then either ‡X.Y, or ‡X.y. Similarly, if ‡x, then either ‡x.Y, or ‡X.y. But either ‡X, or ‡x. Hence, under all circumstances, one of the four complexes X.Y, X.y, x.Y, x.y, *must* be present, and only one can be present. Why only one? Because if we could have X.Y and X.y present together, we should have Y and y present together, which we know to be impossible. Hence, whatever the two events X″ and Y″ may be, we have always $‡_1$(X.Y, X.y, x.Y, x.y), expressed with the counters by ranging them thus (compare art. 21),

X.Y	X.y	x.Y	x.y

These four complexes may be readily remembered from the four compounds AE, Ae, aE, ae, of which the meaning and laws are, as we know, totally different; for we have learned that all those compounds may be present, and that at least *two* of them *must* be present, to fulfil the general conditions (arts. 12, 13, 14). It is important that this resemblance (but not identity,) in form and difference in meaning should be thoroughly well understood.

55. Proceeding in the same way, we see that if ‡X.Y, then either ‡X.Y.Z, or ‡X.Y.z, and so on for each of the complexes of two truths. Hence, as regards complexes of three truths, we must have $‡_1$(X.Y.Z, X.Y.z, X.y.Z, X.y.z, x.Y.Z, x.Y.z, x.y.Z, x.y.z). And so on for complexes of any number of truths, the mode of formation being precisely the same as for compounds of any number of names. The teacher, in developing this set of complexes, will, of course, proceed with counters, in a manner precisely similar to that explained in reference to compounds in art. 17.

56. Now observe that, as only one in each of these sets of complexes can be present, if we know that one *is* present, we know that all the rest are absent. Thus, if the events X″ and Y″ are compatible, that is, if both the assertions X′ and Y′ are consistent, then ‡X.Y, and †(X.y, x.Y, x.y), or, the assertion X′ is inconsistent with y', and both x' and y' are altogether false. Thus, if it is true that *John called on Richard* (X″), and *dined with him* (Y″), it would be inconsistent to say *John called on Richard* (X″), and *did not dine with him* (y''), or that *John did not call on Richard* (x''), even if nothing is asserted about dining, or that *John did not dine with Richard* (y''), even if nothing is asserted about calling. Illustrate this case by a variety of similar simple examples of every-day life.

57. If we know that one or more of several such complexes are present, we know for a certainty that all the others are absent. Thus, if we know $‡_1$(X.Y, x.y), that is, that either both the

events X'' and Y'' happen together, or neither of them happen, then we know $\dagger(X.y,\ x.Y)$, that is, that neither of them can happen singly. Hence, to say that they will either happen together or not at all, is precisely the same thing as to say they will neither of them happen singly.

58. If we know that any one or more complexes are absent from a set, we know that one, and only one, of all the rest must be present. Thus, if we know that X'' and Y'' cannot happen together, or $\dagger X.Y$, we know that $\ddagger_1(X.y,\ x.Y,\ x.y)$, that is, that X'' and y'' will happen together, or else x'' and Y'', or else x'' and y''; that is, that X'' will happen alone, or Y'' alone, or neither X'' nor Y''. If it is not true that *John called on Richard* (X''), and *dined with him* (Y''), then either *John called on Richard* (X''), and did *not* dine with him (y''), or else John did *not* call on Richard at all (x''). Again, if we also know $\ddagger X$, or that John *did* call on Richard; then, as the only complex in which X occurs is $X.y$, we know $\ddagger X.y$, or that John called on Richard, and did *not* dine with him, and at the same time we know $\dagger(x.Y,\ x.y)$. Again, if $\ddagger Y$, or John *did* dine with Richard; then, as the only complex in which Y occurs is $x.Y$, we know $\ddagger x.Y$, or that John dined with Richard, and did *not* call on him. Generally, then, the expression $\dagger X.Y$, which involves $\ddagger_1(X.y,\ x.Y,\ x.y)$, means that if $\ddagger X$ then $\ddagger y$, or if $\ddagger Y$, then $\ddagger x$. Or, as it is frequently put, if X'' happens, Y'' does not happen, or if Y'' happens, X'' does not happen. The proper expression for either of these *hypothetical* assertions is therefore $\dagger X.Y$.

59. It is convenient to take the whole series of complexes, $X.Y$, $X.y$, $x.Y$, $x.y$, and study the 14 results due to the suppositions that first one, then two, and then three are absent, with the usual expressions for these cases, which the above explanations will make easy to the teacher, but which must be carefully *extracted from* the pupil, not *put into* him. All should be worked with *counters* first. Writing for registration is a later, but important stage.

(a.) $\dagger X.Y$ implies $\ddagger_1(X.y,\ x.Y,\ x.y)$, "$X'$ and Y' are not true jointly," "X' alone or Y' alone is true, or both X' and Y' are false," "X' alone or Y' alone, or both X' and Y' together, are false," "if X' is true, Y' is false", "if Y' is true, X' is false." Nothing can be concluded from the supposition that X' is false, or that Y' is false.

(b.) $\dagger X.y$ implies $\ddagger_1(X.Y,\ x.Y,\ x.y)$; "if X' is true, Y' is true," "if Y' is false, X' is false." Nothing can be concluded from the supposition that X' is false, or that Y' is true.

(c.) $\dagger x.Y$ implies $\ddagger_1(X.Y,\ X.y,\ x.y)$; "if X' is false, Y' is false," "if Y' is true, X' is true." Nothing can be concluded from the supposition that X' is true, or that Y' is false.

(d.) $\dagger x.y$ implies $\ddagger_1(X.Y,\ X.y,\ x.Y,)$ "X' and Y' are not both false," "X' alone, or Y' alone is true, or both X' and Y' are true," "if X' is false, Y' is true," "if Y' is false, X' is true." Nothing can be concluded from the supposition that X' is true, or that Y' is true.

(e.) $\dagger(X.Y,\ X.y)$ implies $\ddagger_1(x.y,\ x.y)$; "X' is false."

(f.) $\dagger(X.Y,\ x.Y)$ implies $\ddagger_1(X.y,\ x.y)$; "Y' is false."

(g.) $\dagger(x.Y,\ x.y)$ implies $\ddagger_1(X.Y,\ X.y)$; "X' is true."

(h.) $\dagger(X.y,\ x.y)$ implies $\ddagger_1(X.Y,\ x.Y)$; "Y' is true."

(i.) $\dagger(X.Y,\ x.y)$ implies $\ddagger_1(X.y,\ x.Y)$; "X' and Y' cannot be both true or both false," "either X' alone is true, or Y' alone is true," "if X' be true or false, then Y' is false or true respectively; and if Y' be true or false, then X' is false or true respectively." Compare this with (a) and with (d), and remark the differences. Observe that it is common to say, "either X' or Y' is true," which is ambiguous and might mean either (a) or (i); or else to say, "either X' or Y' is false," which is also ambiguous and may mean (d) or (j). Remember that different eminent logicians interpret this ambiguous phrase differently, and hence, when it occurs, work out both cases separately. The results will always be very different.

(j.) $\dagger(X.y,\ x.Y)$ implies $\ddagger_1(X.Y,\ x.y)$, "X' and Y' are both true or both false together," "X' and Y' stand or fall together," "neither X' nor Y' can be true or false separately;" "if X' is true, Y' is true, and if X' is false, Y' is false, and if Y' is true, X' is true, and if Y' is false, X' is false."

(k.) $\dagger(X.y,\ x.Y,\ x.y)$ implies $\ddagger X.Y$; "X' and Y' are true together."

(l.) $\dagger(X.Y,\ x.Y,\ x.y)$ implies $\ddagger X.y$; "X' is true and Y' is false."

(m.) $\dagger(X.Y,\ X.y,\ x.y)$ implies $\ddagger x.Y$; "X' is false and Y' is true."

(n.) $\dagger(X.Y,\ X.y,\ x.Y)$ implies $\ddagger x.y$; "both X' and Y' are false."

60. From these we see how two or three assertions are sometimes necessary to arrive at a conclusion. Take the *ex absurdo* proof. Suppose it has been proved that "if X' is true, Y' is true," this is (b). From this nothing can be inferred respecting Y. But if we are able also to show that to suppose Y' to be true when X' is false, leads to an absurdity, we can assert that "if X' is false, Y' is false," or (c). Now (c) has been shown to be the same as "if Y' is true, X' is true," and this is the *converse* of (b), which was required. But having both (b) and (c), we have (j), and hence all the results there given, and this is much more than is generally mentioned in an *ex absurdo* proof.

61. Next apply the same sort of analysis to a series of complexes of 3 truths, $X.Y.Z$, $X.Y.z$, $X.y.Z$, $X.y.z$, $x.Y.Z$, $x.Y.z$, $x.y.Z$, $x.y.z$. It will be an excellent exercise to suppose any 2, 3, 4, 5, 6, or 7 of these absent, and determine the meaning of the result. Real assertions should in all cases be invented to suit the abstract cases. These exercises are most conveniently worked with counters. The following are examples.

(a'.) $\dagger X.Y.z$ implies $\ddagger_1(X.Y.Z,\ X.y.Z,\ X.y.z,\ x.Y.Z,\ x.Y.z\ x.y.Z,\ x.y.z)$: Hence, "if X' and Y' are both true, Z is true,"

"if X' is true and Z' false, Y' is false," "if Y' is true and Z' false, X' is false." The pupil should be made to see how this agrees with the theory of *opponent syllogisms* already given, taking X', Y' as the premisses and Z' the conclusion of a syllogism (art. 36). But also "if X' and Y' are both false, or one false and the other true, nothing can be concluded respecting Z'," and this agrees with what was shown respecting the insufficiency of denying the premisses for the purpose of denying the conclusion (art. 35).

(b′.) $\dagger(X.Y.z,\ X.y.z,\ x.Y.z)$ implies $\ddagger_1(X.Y.Z,\ X.y.Z,\ x.Y.Z,\ x.y.Z,\ x.y.z)$; "if or whenever X' or Y' are separately or jointly true, Z' is true," "if or whenever X' and Y' are separately but not jointly false, Z' is true." Nothing can be concluded respecting the truth of Z' when X' and Y' are jointly false. Nothing can be concluded respecting the truth of X' and Y' when Z' is true, but "if Z' is false, both X' and Y' are false."

(c′.) $\dagger(X.Y.Z,\ x.y.Z)$ implies $\ddagger_1(X.Y.z,\ X.y.Z,\ X.y.z,\ x.Y.Z,\ x.Y.z,\ x.y.z)$; "if or whenever Z' is true, either X' alone is true or Y' alone is true;" "if X' and Y' are both true, Z' is false." Nothing can be concluded of the truth of X' and Y' when Z' is false, but "if X' and Y' are both false, Z' is false."

(d′.) $\dagger x.y.Z$ implies $\ddagger_1(X.Y.Z,\ X.Y.z,\ X.y.Z,\ X.y.z,\ x.Y.Z,\ x.Y.z,\ x.y.z)$, "if or whenever Z' is true, X' and Y' are not both false," "if X' and Y' are both false, Z' is false." Nothing can be concluded respecting the truth of X' and Y' from the falseness of Z'.

(e′.) The same process can be extended to any complexes. Thus $\dagger(X.y.Z.W,\ X.y.z.w,\ x.Y.Z.W,\ x.Y.z.w)$ means, "if or whenever X' and Y' are not both together true or both together false, then Z' and W' are also not both together true or both together false," and "if Z' and W' are both together true or false, then X' and Y' are also both together true and false."

(f′.) $\dagger(X.y.Z.w,\ X.y.z.W,\ x.Y.Z.w,\ x.Y.z.W)$ means "if or whenever X' alone or Y' alone is true, then Z' and W' are either both true or both false," and "if Z' alone or W' alone is true, then X' and Y' are either both true or both false."

62. When the pupil thoroughly understands the meaning of such symbols, he will be ready to solve any of the usual questions of this kind, and, in fact, be in a position to undertake problems of much greater complexity. The process he has to pursue is, first to determine how many assertions are made, and to represent them by letters, as X', Y', Z', W'. Then to write out the corresponding list of complexes, for the same number of truths, X, Y, Z, W. Then from the given relations, which will always be expressed in one of the ways just considered, to determine what complexes are *absent*. Each given relation will determine one or more absences, and all the absences must be marked in turn. Then, of the remaining complexes, one, and only one, must be present. It may happen that some simple truth, or some smaller complex, may be present in, or absent from, every one of

the remaining complexes; if so, the corresponding simple or complex assertion is certainly true or false. It is this partial result alone which is contemplated in ordinary logic. It is usual to state these in the form of a syllogism; but this is an unnecessary complexity.

Examples are usually furnished in the singularly inappropriate form, "if A is B, C is D." This is very misleading. It would seem to exclude such cases as "if all the things called A are also called B, then at least one of the things called C is called D;" or "if John comes home to dinner, we will have the sirloin roasted." Again, any such statement as "if X' is true, then Y' is true," is itself an assertion P', which may often, with great advantage, be used as an abbreviation. Thus, take two instances from Fowler's Deductive Logic, p. 114, where they are called *dilemmas*, but they really present nothing peculiar when the above device is adopted.

63. For the first example, let "A is B" be X', and let "C is D and E is F" be Y'. Then any assertion inconsistent with Y', such as "either C is not D, or E is not F," will be y'. Then it is asserted, "if X' is true, Y' is true," or $\dagger X.y$, see art. 59, (b). It is also asserted secondly, "Y' is false" or $\dagger Y$, and consequently $\dagger(X.Y,\ x.Y)$, see art. 59, (f). But (b) and (f) together make up (n), and leave $\ddagger x.y$, or both X' and Y' are false. This is immediately obtained by forming the series of complexes, and seeing that only $x.y$ is left. If instead of simply asserting Y' is not true, we said "neither C is D, nor E is F," or "either C is D, or E is F, but not both," we should still be saying $\ddagger y'$, or some assertion inconsistent with Y' is true. The conclusion will of course be the same. But if the second assertion were simply Y' is true, or $\ddagger Y$, or $\ddagger_1(X.Y,\ x.Y)$ see (h), this would dispense altogether with the first assertion $\dagger X.y$, (which it implies, art. 57,) and give no information respecting X.

64. In the second example, let "A is B, or E is F, but not both" be X', and let "C is D" be Y'. Then the assertions are, "if X' is true, Y' is true," or $\dagger X.y$; and "X' is true" or $\ddagger X$ or $\ddagger_1(X.Y,\ X.y)$, in which $X.y$ cannot be $\ddagger$ owing to the first assertion, so that the result is $\ddagger X.Y$, or both X and Y are true. If the second assertion had been "C is not D" or "Y' is false," that is $\dagger Y$, or $\dagger(X.Y,\ x.Y)$, this, in conjunction with the first gives $\ddagger x.y$, or both X' and Y' are false.

65. The next two examples are also from Fowler, p. 115, and also called *dilemmas*, according to his statement. I have, however, at once put them into the present notation, and will for an example pursue the method of developing the complete series of complexes. For brevity only one series of complexes is used for both examples. When the sign $\dagger$ is placed on the left it refers to the first example, and when placed on the right to the second. The sign $\dagger$ must, of course, be considered as omitted till the work begins. The reader should copy out these complexes, and add the sign $\dagger$ as it is required in the process of work.

$X.Y.Z.W\dagger$	$\dagger X.y.Z.W\dagger$	$x.Y.Z.W\dagger$	$x.y.Z.W$
$\dagger X.Y.Z.w\dagger$	$\dagger X.y.Z.w\dagger$	$\dagger x.Y.Z.w\dagger$	$\dagger x.y.Z.w\dagger$
$X.Y.z.W\dagger$	$\dagger X.y.z.W\dagger$	$\dagger x.Y.z.W\dagger$	$\dagger x.y.z.W$
$X.Y.z.w$	$\dagger X.y.z.w\dagger$	$\dagger x.Y.z.w$	$\dagger x.y.z.w$

66. First, "If X′ is true, Y′ is true," or $\dagger X.y$,
"if Z′ is true, W′ is true," or $\dagger Z.w$,
"X′ and Z′ are not both false," or $\dagger x.z$.

Work.—Take the marks to the left only of the complexes in art. 65.
First $\dagger X.y$, makes the second column †;
next $\dagger Z.w$ makes the second line †;
next $\dagger x.z$ makes the two last complexes in the two last lines absent.

There remain 5 complexes, none of which contain $y.w$, hence the conclusion $\dagger y.w$, or "Y′ and W′ are not both false." Observe that Fowler gives the third assertion and the conclusion in the forms, "either X′ is true or Z′ is true," and "either Y′ is true or W′ is true." These are ambiguous forms, see art. 59 (i), but according to Fowler's definition, p. 113, he means by them $\dagger(X.Z,\ x.z)$ and $\dagger(Y.W,\ y.w)$. Now it is readily seen by this example that the addition of $\dagger X.Z$ would only make the first complex † also, and would leave the complexes $X.Y.z.W$, and $x.Y.Z.W$, so that Y′ and W′ would be true together whenever X′ and Z′ were true separately, and therefore Fowler's conclusion is faulty.

67. Secondly, "if X′ is true, Y′ is true" or $\dagger X,\ y$ } as before
"if Z′ is true, W′ is true" or $\dagger Z,\ w$ }
"Y′ and W′ are not both true," or $\dagger Y.W$.

Work.—Take the marks to the right only of the complexes in art. 65. $\dagger X.y$ and $\dagger Z.w$ make the second column and second line absent as before. And $\dagger Y.W$ make the first and third complexes in the first and third columns absent. There remain 5 complexes in none of X.Z occurs. The conclusion is, that $\dagger X.Z$ or "X′ and Z′ cannot both be true." Observe that as before, Fowler makes his second assertion $\dagger(Y.W,\ y.w)$ and his conclusion $\dagger(X.Z,\ x.z)$, but the addition of $\dagger y.w$ will only make $\dagger x.y.z.w$ in addition, and leave two complexes $x.Y.z.w$, $x.y.z.W$, so that "X′ and Z′ will both be false when Y′ alone or W′ alone is true." Fowler's conclusion is consequently again faulty.

68. These remarks serve to show the importance of attending to ambiguities of language even in formally stated assertions. When they are not stated formally, the utmost caution is necessary. It is quite common to assume $\dagger(X.Y,\ x.y)$ when in fact this assertion depends upon some other, or when further consideration will show from the nature of the assertions made that either $X.Y$ or $x.y$ may be present. "John is either a knave or a fool," is either "John is a knave" (X′) or "John is a fool" (Y′), and is ambiguous, meaning $\dagger x.y$, or $\dagger(X.Y,\ x.y)$, and the latter is usually assumed, although knavery and foolishness (even insanity) are compatible. "John must either win or lose," is either "John will win" (X′) or "John will lose" (Y′), and is usually taken as $\dagger(X.Y,$

$x.y)$. But in the first place John may not play, and in the second place the game may admit of being drawn, so that, unless some other condition is annexed, $x.y$ is possible.

These remarks apply to the old story cited by Fowler, p. 116, which is amusing and instructive enough to reproduce and complete:—"Protagoras, the sophist, is said to have engaged with his pupil, Euathlus, that half the fee for instruction should be paid down at once, and the other half remain due till Euathlus should win his first cause. Euathlus deferred his appearance as an advocate, till Protagoras became impatient, and brought him into court. The sophist then addressed his pupil as follows: 'Most foolish young man, whatever be the decision, you must pay your money; if the judges decide in my favour, I gain my fee by the decision of the court, if in yours by our bargain.' This *dilemma* Euathlus *rebutted* by the following: 'Most sapient master, whatever be the decision, you must lose your fee; if the judges decide in my favour, you lose by the decision of the court; if in yours, by our bargain, for I shall not have gained my cause.'" Now as this is clearly a logical exercise of the old school, and not a fact, it is lawful to invent a conclusion, as follows: "Whereupon the Chief Judge frowned and said: 'Why waste my time? Protagoras had no ground for action till Euathlus had gained a cause. I enter a non-suit. There has been no action. Neither wins and neither loses a cause. But both pay costs.'"

The full statement of every assertion here made is rather lengthy, but the ground of the Judge's decision is easily made out. Let the event,

"Euathlus pays Protagoras" be X″,
"Euathlus gains a cause" be Y″,
"Euathlus loses a cause" be Z″.

Then "if Y″ happens, X″ happens," or $\dagger x.Y$,
"if Z″ happens, X″ does not happen," or $\dagger X.Z$,
"if Y″ does not happen, X″ does not happen," or $\dagger X.y$;

while "it is not possible that Y″ and Z″ should happen together," or $\dagger Y.Z$, although "it is possible that neither Y″ nor Z″ should happen," either by Euathlus not appearing in court at all, or only appearing in a non-suit. Developing the series of complexes, and placing the † as indicated by these assertions, we have

$\dagger X.Y.Z$	$\dagger X.y.Z$	$\dagger x.Y.Z$	$x.y.Z$
$X.Y.z$	$\dagger X.y.z$	$\dagger x.Y.z$	$x.y.z.$

Conclusion $\ddagger_1(X.Y.z,\ x.y.Z,\ x.y.z)$, one of three things must happen: Euathlus gains and does not lose a cause, and pays Protagoras, or Euathlus does not gain a cause (either by losing, or not losing, as in a nonsuit) and does not pay Protagoras, and, as the Judge declared that $\ddagger y$ and $\ddagger z$, (or that Euathlus neither lost nor gained,) there remains only $\ddagger x.y.z$, so that Protagoras was unpaid.*

* Since the lecture was delivered, I have read the original of the story in Aulus Gellius, 5, 10, and am sorry to report that his judges were not quite so sensible as my chief judge; we read: "Tum judices dubiosum hoc inexplicabileque esse, quod utrimque dicebatur, rati, ne sententia sua,

Moral. Add a proviso limiting your bargain as to time. Why did not Protagoras say that he must be paid in five years at any rate, and sooner if Euathlus won a cause sooner? Do not object to the apparently superfluous provisoes in law deeds. Mind that your agreements are unambiguous and distinct. Take warning by the Washington Treaty on the Alabama claims! Remember that "a treaty" is not necessarily nor generally an "amicable arrangement" (for example, that between Germany and France, 26th Feb., 1871); and hence be prepared for at least one side assuming that it is not, although the other side may have meant it to be so. Eschew "negotiations by understanding," and above all things, so far as in language lies, avoid ambiguity. (Compare arts. 66, 67.)

69. Very often the complete development of all the complexes may be saved by some very simple considerations, as in the following, arranged from p. 29 of De Morgan's First Notions of Logic, (8vo., pp. 32, London, 1840). Let there be six assertions, X′, Y′, Z′, P′, Q′, R′, such that ‡$_1$(X, Y, Z), ‡$_1$(P, Q, R) and †X.*p*, †Y.*q*, and †Z.*r*. Show that †*x*.P, †*y*.Q, †*z*.R. Now as there are 6 truths, the complete series would contain 2 . 2 . 2 . 2 . 2 . 2 = 64 complexes, which would be troublesome to write out. But the conditions are immediately seen to be satisfied only by ‡$_1$(X.*y*.*z*.P.*q*.*r*, *x*.Y.*z*.*p*.Q.*r*, *x*.*y*.Z.*p*.*q*.R). For a complex containing P cannot contain Q or R, in virtue of ‡$_1$(P, Q, R), and hence can only contain *q* and *r*; but by †Y.*q* and †Z.*r*, if it contains only *q* and *r*, it can only contain *y* and *z*, and not Y and Z, (art. 59, b). Now in virtue of ‡$_1$(X, Y, Z), if it does not contain Y and Z it must contain X. This gives the first complex, and precisely the same considerations give the others, and show that these are the only complexes possible. The conclusion †*x*.P, †*y*.Q, †*z*.R results from inspection of these results.

70. This concludes all I can venture to say on Deductive Logic for Children.* My tale has been much longer than I designed, but I felt that a mere oral exposition of the method proposed would have been barely comprehended when heard, and would have been forgotten immediately. To have a chance of doing any good, therefore, I was obliged to furnish something approaching to a Teacher's Guide—to be carefully concealed from the pupil,

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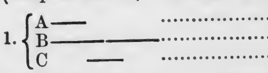
utramcumque in partem dicta esset, ipsa sese rescinderet, rem injudicatam reliquerunt, causamque in diem longissimum [so far off as never to occur] distulerunt." This was a cowardly proceeding, but no decision having been given, we have ‡*y* and ‡*z*, and hence ‡*x*.*y*.*z* as before.

* Many of the paragraphs and notes are for the use of teachers only. They should remember that those distinctions of the old logic which depend upon views of nature have been superseded by the results of the more accurate observations and experiments of modern induction. For deductive logic we have to *assume* that each individual object is as "clearly" and "distinctly" determined by *known attributes*, as a written word is by its visible letters. The "individual conception" thus formed is altogether ideal, and never exists in any concrete research (see art. 80). Hence the distribution of such individual conceptions among "categories," or ultimate classes, is also purely ideal. The famous ten of Aristotle are merely collections of rude conceptions, based on common language, and not on an accurate investigation of distinctions. Even the four of Mill (*feelings*, *minds* experiencing and *bodies* exciting those feelings, with *relations* of succession, co-existence, likeness and unlikeness, between those feelings,) must be regarded as tentative rather than final. Actual conceptions are never ideally or absolutely, but merely *relatively*, "clear" (sufficient to distinguish their objects from all others) and "distinct" (sufficient to distinguish all the parts of any one from all the parts of any other conception). An *attribute* becomes simply a *mark* (as in the old *nota*, τεκμήριον), in the sense of art. 40.

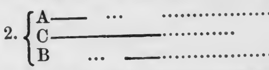
Let both the conceptions A and B be of the class C; then, by art. 44, i., p. 43, †A*c* and †B*c*, and on developing the resultant we have, since ‡(AC, *ac*, BC, *bc*),

$2_2 2_1$ABC, 2_1A*b*C, 2_2*a*BC, ?*ab*C,
†ABc, †A*bc*, †*a*B*c*, ‡*abc*,

(compare art. 24). This gives the diagram 1, where all the different ambiguities are clearly shown by the blank spaces. This most general case, where none of the limitates or doubtful compounds are determined,

1. 

so that it is not certain whether there are or are not any conceptions besides A or besides B which are included in the class C, has not been distinctly contemplated by the old logic. The sub-varieties ‡†A*c* with †B*c*, or †A*c* with ‡†B*c*, have also been omitted. The case really first taken into consideration was ‡†A*c* with ‡†B*c*, giving diagram 2; compare the observations on the predicate, p. 44, note. In this case the conceptions A

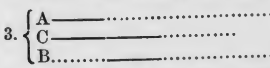
2. 

and B were said to be *co-ordinated*, and each of them to be *sub-ordinated* to the conception C, which was in turn said to be *super-ordinated* to them. The resultant is

$2_2 2_1$ABC, $2_4 2_1$A*b*C, $2_3 2_2$*a*BC, $2_4 2_3$*ab*C,
†AB*c*, †A*bc*, †*a*B*c*, ‡*abc*,

which is more determinate than before on account of ‡(*a*C, *b*C); but there are four compounds still left in doubt, concerning which we can make further suppositions.

Let then ‡†AB in addition to ‡†A*c* and ‡†B*c*, so that not one of the conceptions A is subordinate to any one of the conceptions B, and conversely not one of the conceptions B is subordinate to any one of the conceptions A. Observe that, as ‡*abc* gives ‡*ab*, we could not assume †AB simply, as that would leave ?*ab*, or assume ††AB, as that would give †*ab*; so that ‡†AB was the only assumption involving †AB that could possibly be made. The effect of this is to make †ABC, and consequently, through the limitates 2_1 and 2_2, to make ‡A*b*C and ‡*a*BC, and then again, through the limitates 2_3 and 2_4, to leave ?*ab*C, as in diagram 3, which is formed from the last by filling it up to meet this case. The conceptions A and B are

3. 

now *entirely separate* or *incompatible*; but they remain *co-ordinate* and *subordinate* to the conception C. This case is not particularly named in the old logic, but A and B may be called *incompatibly co-ordinate*. The resultant is

†ABC, ‡A*b*C, ‡*a*BC, ?*ab*C,
†AB*c*, †A*bc*, †*a*B*c*, ‡*abc*,

whom it could only worry and confuse. I think that if any teacher will have the courage to read my lecture in its extended form, and then work through some treatise by its aid, he will find much clear which was formerly dark, and at least emerge from Cimmerian gloom into a kind of twilight.*

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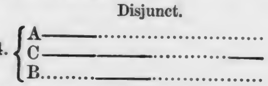
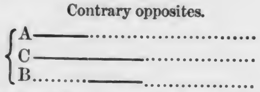
so that there is still one doubtful compound, ?*ab*C, to be determined. If we take ‡*ab*C, there will be part of the conception C which is not covered by any part of either A or B, and then the conceptions A and B are said to be *disjunct*, being incompatible and subordinate to C, but not filling up the whole of C (see diagram 4). If, however, we take †*ab*C, then A and B are *contrary opposites*, being incompatible, but filling up the whole of C (see diagram 5).

Disjunct.

4. 

Contrary opposites.

5. 

It should be mentioned that the diagram for contrary opposites, which is given in Lindsay's *Ueberweg*, is quite fallacious.

Without reference to co-ordination, we have ‡†A*c* for the *sub-ordination* of A to C, and *super-ordination* of C to A; ††A*c* for the *equipollence* of the conceptions A and C, answering to the identity of objects (p. 41, v.); ††AB for *contradictory opposition*; ‡(A, B), that is, ‡AB · ‡A*b* · ‡*a*B · ?*ab* (see p. 17, note, vii.), for *intersection*, where each conception has some part in common, and also some part not in common with the other; ‡†AB for *incompatibility*, as distinguished from contradictory opposition. These are, however, very imperfect representations of the twenty-six assertions in arts. 13, 14, and note, applied to conceptions.

In order that two conceptions A and B should be *disparate*, it is necessary that they should not be both subordinate to any conception except that of an object of thought. Let C be any conception with this exception. The possibility of having either of the conceptions A or B equipollent with C (or non-C), or of both A and B being subordinate to C (or non-C), is excluded by excluding the pair †A*c*·†B*c* (or the pair †AC·†BC, respectively); and these exclusions, whatever be C, are the conditions of disparity. Hence the only possible disparate combinations are the twenty-three other pairs,

made up of one of	‡‡AC,	‡†*a*C,	‡†*ac*,	†AC,	†A*c*,
and one of	‡‡BC,	‡†*b*C,	‡†*bc*,	†BC,	†B*c*.

Of these, the two †AC·†B*c* and †A*c*·†BC give †AB, by art. 23, [1], implying complete separation; while any one of the other twenty-one pairs will give either ‡AC · ‡BC or else ‡A*c* · ‡B*c*, implying intersection in either C or *c*.

For all these cases, which, though difficult to understand in the abstract language of the old logicians, are simple enough when reduced to the preceding form, see Lindsay's *Ueberweg*, pp. 130—135.

* Instead of reading this lecture, a mere oral explanation of the nature of the processes was given, and one or two syllogisms, with the example in art. 68, were worked out with the counters. The whole lecture, ending with art. 70, was printed in the *Educational Times* for June, July, and August, 1872. Several paragraphs and foot-notes have been added in this reprint, and the whole has been carefully revised.

PART II. INDUCTIVE LOGIC.

71. On revising for press the preceding explanation of a Method of teaching Logic to Children, by means of words and counters, it became clear to me that I ought not to content myself with the passing allusion to induction there made (art. 5), but that I should endeavour to show how induction could be taught to children, at least so far as their limited knowledge and capacities allow.* In attempting now to do so, my language must be, as before, addressed to teachers; and, from the nature of the subject, rather consist in an exposition of the general principles which they will have to inculcate, than a precise indication of any method which they can pursue. Hints will be given as I proceed, but details would require a course of instruction rather than a single lecture.

72. In deductive logic we dealt with heaven-sent assertions. These were assumed to be always correct, or rather all inquiry into their correctness was tabooed. All we had to do was to ascertain precisely what they affirmed, denied, and left in doubt, separately and jointly. With regard to separate assertions, attention was confined to the most elementary and most frequently treated, illustrated by assertions respecting the occurrence of certain letters in certain groups of words (arts. 7 to 20). The combined action of assertions was illustrated by the common syllogism, where the premises also related at first to letters in words (arts. 21 to 39), but were afterwards made more general (arts. 40 to 47), and also by an inquiry into the consistency of assertions of all kinds. The very large class of numerically definite assertions, whether of the first kind (art. 48), or of a statistical nature, were passed over as too difficult for children,—the latter were indeed not even mentioned.† The class of assertions respecting mere combinations or juxtapositions of objects was also passed over (art. 48). And yet numerically definite assertions, statistical conditions, and juxtapositions play a great part in all strict inductions. Moreover another class of assertions relating to order or succession, in inference, in space, in time, in action, was not even alluded to; and such assertions have been, on the whole, so little treated by logicians, who for the most part were not already, and were not qualified by their previous studies to become, inductionists, that even an elementary treatise upon them would be an effort of human genius. Yet induction depends greatly upon a

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* The sources of inductive philosophy are the treatises of Bacon and Descartes, and the principal modern works are by Comte, Whewell, Mill, and Spencer. The teacher will find useful compendiums in Fowler's and Bain's Inductive Logic.

† Except in the note to p. 49.

proper method of handling such assertions.* Again, the abstention from all inquiry into the correctness of assertions conditioned an abstention from any inquiry into the probability of their correctness. Yet in the greater part of induction, even in the main facts on which induction and the verification of induction depend, every assertion has only a greater or less degree of probability, of which even the approximate evaluation presents mathematical difficulties not yet in all cases satisfactorily overcome. Moreover, even if we assume that the main facts can be stated, and the conclusions verified with absolute correctness, the method of reaching those conclusions occasionally involves deductions of a nature so much more complicated than any which I have ventured to adduce, that their mere statement would be utterly unintelligible except to persons who had already undergone special and laborious training. It will suffice to say that in order to reduce to a condition of verifiability the most complete and general induction with which we are acquainted, the law of gravitation, it was necessary to invent an entirely new branch of mathematical analysis,† and for its subsequent verification so far as that has hitherto proceeded, other and more delicate processes were initiated by Laplace, and carried out by men of the highest mathematical genius. But inquirers even now find themselves stopped by insuperable difficulties in many special domains, as for example, in hydraulics. In other branches of natural science we are much less capable of arriving at exact verification; and it has been the glory of such a man as G. S. Ohm, who has only lately died, to indicate, and of such a man as Helmholtz, who still lives, to verify approximatively, inductions respecting such extremely elementary and long considered subjects as the sounds of music, on which Pythagoras himself is reported to have dogmatized.

73. With our few and easily stated assertions in that portion of deductive logic which I previously considered—and I warned you that I was only scraping the soil of a great subject (art. 3)—it was very easy to arrive at clear, precise, and certain results. This is entirely changed in induction. All our data are more or less hazy; and hence our conclusions must share the same fate. The word induction runs so glibly off the tongues of most speakers, that I may seem to have been raising giants for the purpose of slaying them. But unfortunately I found the giants already there: and I am quite unable to slay them. The only comfort for our present purpose is, that even if a steam *mitrailleuse* method of slaughtering these giants wholesale had been perfected, school children would be quite incapable to manage it, and that if we are content merely to show the nature of the method by rough instances which a child can manipulate, we can skirt the fastnesses of these ogres, and make a real advance into their country, on a road suited for infant feet. In fact, as we shall see, much may be

* The (imperfect) method in which such assertions of succession are reduced to assertions of inconsistency is illustrated below (art. 81).

† Newton's "Prime and Ultimate Ratios," subsequently known as "Fluxions," and now generally replaced by some form of "Differential and Integral Calculus."

done without any systematic knowledge of even deductive logic (art. 84). What I am now anxious to do is to show how teachers may put young minds upon what to present science appears the right track, thus saving them from dogmatism upon insufficient data on the one hand, and from scepticism through despair of all knowledge on the other; and leading them to feel that when the truth or real law evades us, we can and must put up with a contrivance, a theory, which we are convinced is not the reality, but which we must treat as such, till the means are at hand for correcting it,—they probably never will be at hand for perfecting it,—valuing it as the most trusty attainable weapon, but ready to reject it at once if it snaps with any blow, however unexpected; or, at least, to limit its use to less adamantine obstacles, and leave the others unassailed.* We have, indeed, constantly to be content with a statement of laws which we are convinced are *not* correct, and to argue from them as if they *were* correct, in order to discover the limits within which they may be used with safety. This is, in fact, our position with regard to almost all physical, biological, social, and moral science. Our knowledge is all on its trial, but is not the less valuable; for *wherever we can predict results with certainty, our theory, so far, if not true, is at least as good as true.* Some metaphysicians are inclined to think that man has never got, and can never get, beyond this. Whether they are right or wrong is of very little consequence. It is quite enough to have got so far.

74. We scarcely ever speak without making an assertion. Deduction, as we have seen, dealt with the meaning of assertions already made. *Induction deals with the method of making verifiable assertions.* It is easy to make assertions right and left, if we are never called to account for them. Theodore Hook described the pleasure felt by a barrister who had turned parson, that, when he preached,

* We shall thus escape the error of those "who set up *their own conceptions* of the orderly sequence which they discern in the phenomena of nature as fixed and determinate *laws*, by which those phenomena not only *are*, within all human experience, but always *have been*, and always *must be*, invariably governed," stigmatised by Dr. Carpenter in his Presidential Address to the British Association at Brighton, 14th August, 1872. This very widely circulated essay contains many illustrated allusions to the matters here treated, in a form having especial reference to children in science as well as adults—the Associates as well as the Members of the British Association. Attention will consequently be frequently drawn to it in future notes. The present lecture had been completed a fortnight before Dr. Carpenter's address was delivered; and nothing could have been more unexpected than such an address on such an occasion. But the necessity for something like scientific instruction in schools was well illustrated on this occasion, for the Lord Lieutenant of the county, the chairman of an educational society in high repute, in proposing the usual vote of thanks, seemed to take it as a matter of course (according to the report in the *Brighton Daily News* for 15th August, 1872, p. 10, col. 1) that outsiders should know nothing about such elementary matters, saying, "I should like to suggest to *many* of the friends here present, who perhaps, *like myself*, *have not had a very scientific training*, that the address is one which will amply repay a second or a third perusal. *A good deal of it, I must confess, I did not at first quite understand.*"

there was no longer any one to get up on the other side. But the man of science is his own opponent. He is not satisfied with his own assertions till he has run them to earth, deduced from them as many of their consequences as he can (that is, as we now know, discovered their meaning separately and jointly so far as possible), and contrived means of contrasting these consequences with fact. To him *verifiable knowledge is the only knowledge.* He is very well aware, however, from sad experience—the verification requires no coaxing—that many of his theories are non-verifiable, often as to bare possibility; but he at least takes care never to call *them* knowledge. In this respect he differs widely from the metaphysician, the intuitional philosopher,* who frequently contends that he is as certain as he is of his own existence that some unverifiable theory represents a fact. Such theories are not without a certain value, and sometimes a very high value; but they are never knowledge; they are not inductions, and they do not belong to our present subject, except so far as it is necessary to warn young inductionists that even induction is not everything, and that our world would be a poor world indeed if it only dealt with what happens at the present moment to be verifiable, and that, as the admission of verification into the arena of science is scarcely more than two or three hundred years old, and as our means of verification have largely and rapidly increased as its necessity has been more and more felt, we ought not to be too hasty in determining the limits of future verifiability.† But this is a question for the master, not the learner, of his art, and we must strictly limit our school inductions to what is apparently verifiable.

* Dr. Carpenter says, "By the *intuitionalists* it is asserted that the tendency to form these primary beliefs ['which constitute the groundwork of all scientific reasoning,' as previously defined] is inborn in man—an original part of his mental organisation; so that they grow up spontaneously in his mind as its faculties are gradually unfolded and developed, requiring no other experience for their genesis than that which suffices to call these faculties into exercise. . . . *The intellectual intuitions of one generation are the embodied experience of the previous race.* . . . It appears to me there has been a progressive improvement in the *thinking power* of man. . . . As there can be no doubt of the hereditary transmission in man of acquired constitutional peculiarities, which manifest themselves alike in tendencies to bodily and to mental disease, so it seems equally certain that *acquired mental habitudes* often impress themselves upon his organisation with sufficient force and permanence to occasion their transmission to offspring as *tendencies to similar modes of thought.*" This doctrine, as Dr. C. mentions, was "first explicitly put forth by Mr. Herbert Spencer." Miss F. Power Cobbe, in her essay called *Darwinism in Morals*, combats Mr. Spencer's statement.

† At the Brighton meeting of the British Association, on Monday evening, 19th August, 1872, Prof. W. K. Clifford delivered a lecture on "The Aims and Instruments of Scientific Thought," which has not obtained so wide a circulation as the President's address, but is of extreme value to any persons who wish to obtain a correct notion of scientific thought. It has been printed in *Macmillan's Magazine* for October, 1872, from which all the citations will be made. "When we are told that the infinite extent of space, for example," says Prof. Clifford, and the same applies to any other existing unverifiability, "is something which we cannot conceive at present, we may reply that this is only natural, since our experience has never yet supplied us with the means of conceiving such things. But then we cannot be sure that the facts will not make us learn to conceive them; in which case they will cease to be inconceivable," (p. 511, col. 2). And speaking of another supposition, he says, "The knowledge of that fact would be different from any of our present knowledge, but we have no right to say that it is impossible." (*Ib.*, col. 1.)

Observe that unverifiable assertions may still be made, and deductive rules applied to them; but the conclusions, as we know, can never have any more weight than the assertions themselves on which they depend. This limitation at once strikes off all supernaturalism, all suppositions of "occult causes," all assumptions of what *may* be because it cannot be shown *not* to be, all hypotheses concerning the existence of fluids, ethers, spirits, atoms and what not, made with any other view than colligating results, and obtaining verifiable laws.* This remarkably simplifies the questions to be considered. Many persons will be induced to conclude that with these limitations "thinking is but an idle waste of thought," but the field will soon be found far larger than man at present can plough, and sow, and harvest. And as far as school instruction is concerned, there is the utmost value in drawing the line thus sharply. It is not quite that between secular and religious instruction, for it cuts off much which is purely secular; but the wide debateable land thus left between induction proper and religion, will serve not only to render the distinction clearer—

* "Electric fluid" is a word of constant occurrence in newspapers. Its existence is now entirely discredited, yet *two* electric 'currents,' a positive and negative, are familiarly spoken of by men of science. The 'luminous ether,' supposed to pervade all space, and to generate, by its various 'modes of motion,' heat and light, if not other physical phenomena, must be regarded as a mere vehicle for obtaining and colligating laws. Its existence has never been verified. The mathematical laws might be the same, if derived from other sources. Nothing is more delusive in this respect than a mathematical formula. Thus, $xy = a^2$, results from a consideration of a being a mean proportional between x and y, from x and y being lengths of the sides of a varying rectangle with a constant area, from x and y being the coordinates of a point in an hyperbola, or the distances of two points from the centre of an involution, &c. "Spirit," in Newton, is an ether, conditioning physical forces. See the last paragraph of the *Principia.* "Adjicere jam liceret nonnulla de spiritu quodam subtilissimo corpora crassa pervadente, et in iisdem latente; cujus vi et actionibus particulae corporum ad minimas distantias se mutuo attrahunt, et contiguae factae cohærent; et corpora electrica agunt ad distantias majores, tam repellendo quam attrahendo corpuscula vicina; et lux emittitur, reflectitur, refringitur, inflectitur, et corpora calefacit; et sensatio omnis excitatur, et membra animalia ad voluntatem moventur, vibrationibus scilicet hujus spiritus per solida nervorum capillamenta ab externis sensuum organis ad cerebrum et a cerebro in musculos propagatis." Such ethereal and spiritual vibrations are only another name for *periodically recurring states*, and that is all which is expressed by the resulting mathematical formulæ. The old "animal spirits" are quite dead. The "spirits of the dead" do not yet belong to science. "Atoms" are familiarly spoken of by chemists, who have formed from them a powerful theory for the investigation of the relations of substances, which we cannot at present replace, but which can only be regarded as provisional; for though the resulting laws can be controlled, the existence of atoms is quite unverifiable.

because distinctions always become confused near the limits*—but to impress very strongly on the pupils' minds that *induction is not the sole sanction to thought recognized among men*, and hence, to save them from a scientific, which is quite as hurtful as a metaphysical or supernaturalistic pedantry. There are several classes of researches where verification is not possible; as, for example, in theories of the origin and treatment of disease, in geological and historical investigations, where we cannot put back the clock of time, and contrast the actual past with its hypothetical aspect. Such studies are admitted as partially positive, because the different results are such as contemporary men might have verified, so that, though unverifiable to the present generation, they are not so to humanity at large. But, inasmuch as they are not absolutely verifiable,† they do not possess the same amount of evidence, and require special and very delicate treatment to be admitted as scientific facts at all. In all cases the power of verifying must not be limited to the inductionist's own powers, but must be extended to all investigators in all time, the main point being that future verification should not exceed the bounds of human power. But until verification *has* taken place, we have, at most, reached probability, and real knowledge, science proper, does not exist.

75. The intention of all the processes called inductive is, *from the known present and known past, to discover the unknown present and past, and to predict the unknown future.* This discovery is, at any rate, scientifically, never designed to gratify idle curiosity, but rather to obtain a basis for further prediction. And this prediction must not be looked upon as a gipsy peep into futurity, but as the only means we have of preparing for contingencies, and advancing in any way, material or moral. It is often said, that if man knew what was to happen, he would be the most miserable of beings. Yet man is always striving to know what will or may happen, and to provide for the rainy as well as the sunny day. If, in winter, we could not look forward to seed-time; if, when sowing, we could not look forward to harvest; our existence would be exhausted in a hand-to-hand fight with death. But we now know how the seed-time can be best utilized by proper prepara-

* "The animal and vegetable kingdom have a debateable ground between them, occupied by beings that have the character of both, and yet belong distinctly to neither. Classes and orders shade into one another all along their common boundary. Specific differences turn out to be the work of time. The line dividing organic matter from inorganic, if drawn to-day, must be moved to-morrow to another place; and the chemist will tell you that the distinction has now no place in his science, except in a technical sense, for the convenience of studying carbon compounds by themselves." (Prof. Clifford, p. 505, col. 2.)

† The geologic induction that there will be a seam of coal found in a certain locality is of course verifiable by digging down. But the geologic theory which accounts for the formation of that seam of coal is not verifiable, as the formation of coal took place in prehistoric ages. The statical and dynamical theories of geology are totally different in the evidence on which they rest. The text especially applies to the latter.

tion of the ground, and how, by commercial intercourse, involving predictions innumerable, we may even supply the dearth of one country's harvest by the plenty of another. The teacher must always be prepared for the old old question, What's the use? But he should resolutely refuse giving a detailed answer. It is enough to know generally that, without a means of predicting, however roughly, life would be an abject, objectless chaos, and that whatever contributes to more accurate prediction, no matter how slight the apparent connection between the discovery and any individual's special needs, is advancing so far the whole human race. *We gain knowledge in order to predict, and we predict in order to provide.** not by or for ourselves individually, but by and for the whole human race at large. And in thus acting for a remote or uncertain future, we must remember that an incalculably greater part of all the knowledge and all the comforts therefrom arising which the present generation enjoys, is due to a remote and uncertain past. It is a text on which the teacher may often be able to preach with advantage.

76. The inductive faculty shows itself in earliest youth, and is certainly developed in the lower animals. Hence, when a child comes to school, he is fully competent to feel the processes through which he may be put, though it would be absurd to lay down the laws in a dull, dry, abstract manner. We must get him to perform from mere habit certain processes with which he has become more or less familiar, but of the meaning of which he is far from being conscious; and then make him feel what he has done, so that he may afterwards apply those processes intelligently. The ability to educe these powers marks the educator. One of the most frequent processes of thought the child goes through, when he sees or hears anything, is to make a guess (English), a supposition (Latin), or an hypothesis (Greek). They are all the same thing. He sees one thing, and immediately looks forward to another, or assumes that some other has gone before. You may call this instinctive, if you will. It is certainly part of our constitution, and it is the foundation of *all inductive reasoning*, which *is merely regulated guessing.*† The child guesses from certainly a

* According to Comte's maxim: "Savoir pour prévoir, afin de pourvoir."

† This is, in fact, the "common sense" spoken of in Dr. Carpenter's address; "the value of its results depending in each case upon the qualifications of the *individual* for arriving at correct results . . . . . the trustworthiness of their common sense decision arises from its dependence, not on any one set of experiences, but upon *our unconscious co-ordination of the whole aggregate of our experiences*—not on the conclusiveness of any one train of reasoning, but on *the convergence of all our lines of thought towards this one centre.*" These words must not be pressed home. True, our judgment at any time is the habitual outcome of all our experience and previous judgments, but that experience and those judgments are not generally present to our mind when making the new judgment. So frequently are important links forgotten, and judgments first formed have to be subsequently modified, that "second thoughts are best," has past into a proverb. It is only so far as this "unconscious co-ordination" becomes "conscious" that trustworthiness results, and then only (though not always) when "the whole aggregate of our experiences" has been large, as in the case of "experts." The real whole aggregate to be consulted is the recorded experience of humanity.

very narrow field of observation and experiment, with, most likely, a very imperfect appreciation of the circumstances presented to his mind, and also generally in a very awkward and round-about manner.* But *the fundamental principle of all reasoning is* merely *to form the simplest supposition which is consistent with the* WHOLE *of the circumstances to be represented;* and what is this but the child's process clarified? All the so-called canons of induction are but evolutions of this one principle (art. 82). The supposition must be the simplest possible, and it must be consistent with every one of the circumstances involved, considered not merely isolatedly, but more especially as a whole, as parts which mutually affect each other. The intentional suppression of some circumstances is scientifically unthinkable, except they be recognized as immaterial, but the accidental suppression of some material circumstances through ignorance is frequent, and hence the necessity for means of detecting them, by observation and, when possible, experiment. Occasions will very frequently arise in the course of instruction or in school life in which the teacher will have to suggest material circumstances which have escaped the observation of the pupil. This is constantly the case where the young reasoner sees no difficulty at all. Without a knowledge of any circumstances, no supposition at all can be made. Hence, so-called *à priori* assertions are misnamed; they are not assertions made without any knowledge, but assertions made with extremely imperfect knowledge, occasioning the employment of inappropriate analogies, and, as we now know, leading generally to worthless results. The *light of nature,* which is a very favourite source of inspiration for quick children, is also powerless in presence of ignorance of circumstances, though it will often help towards the solution of a troublesome problem in a more or less awkward manner, for the light of nature is a lamp which requires trimming with the scissors of experience and plenishing with the oil of observation. It burns however with a very different flame in different minds, and the danger is that children will come to rely upon it without troubling themselves with learning, ignorant that, as has been well said, *genius is above all things a capacity for hard work.*

77. Now the basis for making any supposition concerning circumstances which are presented to the mind, is that the supposer has seen similar circumstances concur, and observed results similar to those which he consequently anticipates. There is no doubt of this fact as regards the supposer, but the question is, what justifies him in this conclusion? And the only answer as as yet given is, the event. A feeling has grown up among men, which is stronger the more opportunity there has been for accurate examination, that *there exist invariable and unconditional relations respecting the succession and co-existence of all circumstances.** It is felt that it is only our ignorance which stands in the way of our predicting with certainty what events will follow after or concur with any given event. But the mode of expressing this feeling is very diverse, and is often accompanied with strange limitations, due to previous ignorance and present prejudice. On these I refrain from entering. If any child start some question of necessity and freewill,—and he may do so, for he may bring his school knowledge home, and his crude statements of the teacher's doctrines may alarm some sensitive parent into a dread lest the child may be taught some "unsound" notions of divine government,—the teacher has only to say: "As you learn more, you will understand these things better. You must learn very much to understand them well, but you know now that if you strike a ball, the ball will move from you, and that you need not strike the ball unless you like. The motion of the ball depends upon the kind of blow you give it. Your liking to strike it

* This last circumstance does not mark the child only. Every investigator is aware that the way in which he is first led to some new conception is generally very obscure, and that the processes he consequently adopts to work it out are subsequently discovered to be most unnecessarily circuitous. Simplicity results from much knowledge and many trials. Success is the outcome of numerous failures.

* "The step from past experience to new circumstances must be made in accordance with an observed uniformity in the order of events. This uniformity has held good in the past in certain places; if it should also hold good in the future and in other places, then, being combined with our experience of the past, it enables us to predict the future, and to know what is going on elsewhere; so that we are able to regulate our conduct in accordance with this knowledge. The aim of scientific thought, then, is to apply past experience to new circumstances; the instrument is an observed uniformity in the course of events. By the use of this instrument it gives us information transcending our experience; it enables us to infer things which we have not seen from things that we have seen; and the evidence for the truth of that information depends on our supposing that the uniformity holds beyond our experience." (Prof. Clifford, p. 502.) "We say that the uniformity which we observe in the course of events is exact and universal; we mean no more than this, that we are able to state general rules which are far more exact than direct experiment, and which apply to all cases that we are at present likely to come across." (*Ib.*, p. 506, col. 1.) "It is possible that by-and-by, when psychology has made enormous advances and become an exact science, we may be able to give to testimony the sort of weight which we give to the inferences of physical science. It will then be possible to conceive a case which will show how completely the whole process of inference depends on our assumption of uniformity. Suppose that testimony, having reached the ideal force I have imagined, were to assert that a certain river runs up hill. You could infer nothing at all. The arm of inference would be paralyzed, and the sword of truth broken in its grasp, and reason could only sit down and wait until recovery restored her limbs, and further experience gave her new weapons." (*Ib.*, p. 506, col. 2.) The principle of the uniformity of nature is the *postulate* of induction, an assumption without which any inductive reasoning would be impossible. We have not yet seen our way to any means of demonstrating its truth *à priori.* Every coincidence of a result with a prediction founded upon it, helps to verify it. But any absence of such coincidence that could not be traced to mere errors of deductive reasoning of observation or of record, would be destructive of its value, would, as Prof. Clifford remarks in the passage just quoted, "paralyze the arm of inference," and leave us ignorant of everything which we are now supposed to know.

or not to strike it, depends upon something else. Thus you may not like to strike it, because you are told to do so, or because you are afraid you cannot make it go far enough, or strike it in the best possible way, or feel ill, or lazy, or 'don't see the use,' and so on. The difference between the ball's motion and your liking to strike it, is so far a difference of simplicity. Let us try to understand the motion of the ball, which you will find useful at fives and at cricket, and which is quite difficult enough for neither you nor me to understand thoroughly, and leave the liking for another time, knowing that at best we shall always understand it very imperfectly."

78. It is necessary that occasion should be taken very early indeed to show that this absolute invariability and unconditionality can never be thoroughly appreciated in the things we see or feel, or deal with, but only in certain imaginary things which we come to conceive by leaving out of consideration all those little peculiarities that are too complicated to allow for.* Thus, in determining the motion of the ball, we should first suppose it quite smooth, and this is not the case in the best cricket ball, much less in a common fives ball; and also that it is perfectly spherical, which we know is never the case; and also that it is equally elastic in all places, which those who have tried to make a fives ball for themselves will know they could never manage. All these things are better obtained in a billiard ball or a glass globe, but these would probably shiver to pieces with a blow of the bat. We must however consider that the ball will never break, nor the bat either, hit we ever so hard. We hit an abstract ball with an abstract bat, and then we can tell tolerably well how the ball would fly, provided there were no air! But we have a real ball, a real bat, and plenty of air, already in motion generally, in the shape of wind, hence the ball never goes exactly as we foretold, but always nearly so, and the more nearly the nearer the realities approach to the abstrations. That is, there are *invariable unconditional relations*, but these *hold in all their perfection, solely for abstractions*, abstract things and abstract events. For realities we have always to make certain allowances.† Why not go to the realities at once?

* Science, therefore, deals with an abstract, ideal world. It is never more than an approximation to reality. This must be carefully borne in mind in weighing the apparently unqualified assertions of men of science, who are so familiar with the fact that they forbear to mention it, as even the ecclesiastical writer neglects putting "D. V." after every future tense, notwithstanding Jas. iv. 13—15.

† The three angles of a triangle, says Euclid, together make up two right angles. Probably no one doubts the fact, although all geometricians know the extreme difficulty occasioned by one of the assumptions (in this case a pure assumption) on which it is based. But try to put it to the test. What man ever drew on paper a triangle which strictly satisfied the condition? Nay more, what surveyor, in making his triangulations, either with his measuring chain, or with all appliances of scientific apparatus, as in national trigonometrical operations, ever found such a triangle? In the latter case, indeed, where the sides of the triangle are many miles in length, the observer always finds the sum of his three angles greater than two right angles, the difference (known as "the spherical excess," because due to the nearly spherical form of the globe) having to be carefully allowed for. If, then, we only knew our geometry by concrete instances, we should be in a state of muddle. It was the happy exercise of man's power of abstraction by the ancient geometers which allowed us to see the invariable and unconditional abstract law through the tangled net of concrete circumstances.

Simply because man's mind is unequal to the task, the abstractions giving him already more than he is able to accomplish. *All we can do is to make the realities as near the abstractions as possible, and allow for the differences.* And in this precept is involved another extremely important principle, setting forth the bounds of human power. *We cannot alter the invariable unconditional relations which determine the order of events, but we can alter the intensity with which the circumstances enter into the events whose order is thus determined.* We cannot prevent the ball starting from the bat when struck, but we can regulate the strength and direction of the blow, and we can abstain from giving the blow altogether. We cannot prevent ignited gunpowder from exploding, but we can keep the heat to so low a degree that ignition will never take place. It really does not require a very great step further to reconcile necessity and free-will, and the teacher will do well to show over and over again, by simple instances, whenever they occur, that *relations are fixed, conditions variable.* The knowledge of these fixed relations, and of the effect of that variability of condition, is, as Bacon long ago expressed it, coincident with human power.*

79. There are a few words which arise out of what has been said, and from being in constant use, and therefore having vague popular senses attached to them, require careful consideration.

i. The first of these is *law*. A *scientific*, or, as it is often called, a *natural law*, has only the one determinate meaning, of *an invariable unconditional relation of succession or coexistence.* Any other meaning is unscientific. The clear and definite statement of such a relation bears a vague resemblance to the written law of the realm, which however can be altered, and can be evaded, and which has been settled by the will of certain legislators. This law of the realm lays down penalties in case it should be disobeyed, but the appointed executive is often powerless to enforce them, and even to detect the disobedience. These laws are therefore in no respect invariable or unconditional. Owing to our power of modifying the intensity of circumstances, we are able to bring things or events within the conditions which render the results of scientific laws appreciable or otherwise; but having brought them within this influence, there is no choice of obedience or disobedience, the invariable relation invariably asserts itself. What then can be meant by saying that a person by over-eating "broke" the natural laws of health, and suffered the "penalty" of disease? The whole sentence is one of confused analogy, which is however very widely employed. The real meaning is, that the person has

* "Scientia et potentia humana in idem coincidunt, quia ignoratio causae destituit effectum."—*Novum Organum*, Aphorism 3.

brought himself under the action of that invariable relation between food and vital functions which results in disease; whereas, if he had not brought himself within that action, this disease would not have occurred. It does not follow that some other disease might not have occurred, for the person might have come within the conditions for the occurrence of that second disease, even if he had avoided the first. There is, however, a little more meant, namely, that the over-eater has put himself beyond the relations which result in health; and it is this voluntary part which apparently gave rise to the whole conception, most widely disseminated by George Combe's *Constitution of Man*, a work which in its day produced a profound impression among a very numerous class of readers.* This should be illustrated by such a simple case as a boy putting his finger too near to any flame. The distance at which he holds it is generally voluntary; but he may fall, or be thrown against the source of heat. It could hardly be said that a law had been enacted against his putting his finger too close to the flame, and that the destruction of the skin, and

* In the introduction to his work on "the Relation between Science and Religion," of which the 4th enlarged edition (here cited) appeared in 1857, a year before his death, Mr. Combe says (p. xxx.) "To prevent misunderstanding, I beg here to explain the meaning which is attached in the following work to the expressions 'Laws of Nature' and 'Natural Laws.' Every object and being in nature has received a definite constitution, and also the power of acting on other objects and beings. The action of the forces is so regular, that we describe them as operating under laws imposed on them by God; but these words indicate merely our perception of the regularity of the action. It is impossible for man to alter or break a natural law, when understood in this sense; for the action of forces, and the effects they produce, are placed beyond his control. But the observation of the action of the forces leads man to draw rules from it for the regulation of his own conduct, and these rules are called natural laws, because Nature dictates or prescribes them as guides to conduct. If we fail to attend to the operations of the natural forces, we may unknowingly act in opposition to them; but as the action is inherent in the things, and does not vary with our state of knowledge, we must suffer from our ignorance and inattention. Or we may know the forces and the consequences which their action inevitably produces; but from ignorance, that through them God is dictating to us rules of conduct; or from mistaken notions of duty, from passion, self-conceit, or other causes, we may disregard them, and act in opposition to them: but the consequences will not be altered to suit ignorant errors or humours; we must obey or suffer." There is, therefore, a constant confusion of the two meanings attributed to natural laws, (1) invariable, unconditional relations, and (2) rules for applying conditions so as to effect desired ends in accordance with those relations. These latter state, "If you do A, you will obtain the desired B, if you do not, you will not obtain B, and if you do X, or Y, or Z, you will obtain the undesired C." And it is very probable that there will be more or less error in the statement of conditions and result, because the generality of the terms in which the conditions are stated leaves out of consideration a vast number of peculiar circumstances which seriously affect the result in particular cases. "To break the law," means "not to use the condition A," or more frequently, "to use one of the conditions X, Y, Z." Upon the "coercive" character of law, see the last foot note to No. iv.

consequent pain arising from too near an approach, were the penalty attached to breaking this law. It becomes evident that there was only a certain definite relation between the amount of heat and its effect on the tissue, and something of the same kind was the case in over-feeding.

ii. This leads to another great difference between a country's laws and scientific laws. The former are carefully promulgated, so that the judge is always entitled to disregard the plea of ignorance of the law. The latter have to be discovered. They are not only in great part unknown still, but are, more correctly speaking, only known to a very small extent indeed, and it is the business of men of science to discover them. What becomes of the notion of penalty attached to breaking an unknown law? But here steps in a very important word, which will have to be well understood. *Chance applies to all those events whose Law is unknown.* The feeling that law is universal is now so generally entertained, that men of science refuse to believe in the absence of law, much less to erect the absence of law into a disposing goddess co-equal with fate. Hence, when they say that a thing happens *accidentally*, or *by chance*, they simply mean that its laws are unknown.* And they point to a remarkable confirmation of this view in the so-called *law of averages*, which shows that when a large number of events of the same kind is taken into account, although we are unable to predict what will be the result for any one in particular, they will all be grouped round one central phenomenon. This should be illustrated. A boy strikes a ball, say in the game of 'trap, bat, and ball.' The number of times which he strikes it, and the distances at which the ball comes to rest, should be noted. (The problem would be a little too complicated if the exact places of rest, instead of merely the distances, were observed.) A distance found by adding all the observed and measured distances together, and dividing by the number of blows

* Dr. Carpenter (*ibid.*), using the flint implements found at Abbeville and Amiens as an illustration, says: "The evidence of *design* to which, after an examination of one or two such specimens, we should only be justified in attaching a probable value, derives an irresistible cogency from accumulation. On the other hand, the *im*probability that these flints acquired their peculiar shape by *accident*"—where the phrase only means by some unknown, and even unguessed relation, different from that previously assigned by ourselves, of having been formed by man as instruments—"becomes, to our minds, greater and greater as more and more such specimens are found; until at last this hypothesis, although it cannot be directly disproved"—that is, although we cannot show directly that our own hypothesis is correct, or cannot verify it—"is felt to be almost inconceivable"—that is, incapable of representing to *our own minds* the *whole* of the circumstances known to *us*—"except by minds previously 'possessed' by the 'dominant' idea of the modern origin of man." This "dominant idea" means a conception formed from what Dr. Carpenter considers insufficient consideration, or on grounds which he rejects, but which the holder maintains to be *one* of the conditions which he cannot ignore, so that to conclude that the flints had been humanly formed would not represent the *whole* of the circumstances which he *regards as known to him*.

struck, would give the "average" distance. It would be found that if this average were determined from a large number of trials, say 100, it would be almost precisely the same for another hundred, provided the same boy made the trial. Also, if 100 boys were to try each three times, the average distance for each set of 100 blows would be found nearly the same. Make trials with tossing a penny;* with picking balls from bags of white and black balls; with walking to a place or touching squares on a chessboard blindfold: with the number of words a boy can read intelligibly, or write legibly in a minute, the number of words in a line of writing or print, and so on. Show the application of this in the average number of persons who travel by different classes in railways or over different lines, by which the officials have to regulate their accommodation; by the average number of things sold in the market, and the average supply brought in, and the average prices at which they are sold. In all examples, point out the large number of influences that there must be at work, but which we are unable to evaluate, and hence the great importance of the

* Tossing a penny is an admirable illustration, and it is worth while spending a quarter of an hour over it, the whole class experimenting and *registering* the results as follows. Let each place the penny head uppermost each time, toss it a little height only, and receive it on a crumpled handkerchief, to prevent rebounding. Number 32 lines on a slate, write *h* for head and *t* for tail, and write in each line the result, till you get to a *h*. There will then be 32 *trials*. The following is the result of 32 actual trials, the commas dividing the lines: *h, h, th, h, tth, h, h, th, h, th, h, th, h, h, h, h, h, h, ttth, th, h, h, h, tttth, tttth, th, th, tttth, h, tth, h, h*. Now as there is an "even chance" of throwing *h* each time, we should expect *h* in 16 of the 32 trials; there are actually 19. As in two throws we might have *hh, ht, th*, or *tt*, it is 3 to 1 against having *th*, and hence we should expect *th* in 1 case out of 4, that is in 8 out of 32, we really find it 7 times in the 32 trials. Again, in three throws we might have *hhh, hht, hth, htt, thh, tht, tth, ttt*, so that it is 7 to 1 against *tth* or *2th* as we may write; we should therefore expect *2th* once in 8 trials, or 4 times in 32, we really find it twice. Similarly for *3th*, we should expect it once in 16 times, or twice in 32 times, we really find it once. Again, *4th* we should only expect once in 32, we find it twice. But *5th* should be only half a time in 32, that is once in 64 trials, we really find it once in the 32. Let the pupils add the results, in a table thus:—

Trials 32;	expected,	found.
h,	16,	19
th,	8,	7
2th,	4,	2
3th,	2,	1
4th,	1,	2
5th,	1,	1
Total trials	32	32

Now this is a very small experiment, and the results are really much closer to the theoretical amounts than we have any right to think would often happen. But if all the results in the whole class are added after registration, we should probably come much nearer. The following table represents the results for *h* and *th* actually found for a very large number of trials, the first 2,048 having been made by the celebrated naturalist Buffon, and are calculated from a table given in Prof. De Morgan's *Budget of Paradoxes*, p. 170 (London, 1872). The first column gives the actual number of trials, the second the numbers of *h* or *th* theoretically expected, the third the numbers of *h* or *th* actually found, the fourth what this would give in 100,000 trials at the same rate, and the fifth the theoretical amount in 100,000 trials, and the sixth the difference between the two last amounts:—

law of averages, and of the observations on which they are based, such as the decennial census.

iii. Draw attention to the fact that this law of averages applies to *voluntary actions* (leading a little further to the solution of the question of necessity and free-will); for notwithstanding the general controlling average, there is no sense of restraint in any actor. This leads to a more accurate notion of *restraint*, which, in voluntary agents, amounts to the presentation of a choice, coupled with the withholding of a choice usually open. We have no

	trials	expected	found	or in 100,000	instead of	difference
h	2048	1024	1061	51806	50000	+ 1806
	4096	2048	2191	51489	,,	+ 1489
	6144	3072	3126	50879	,,	+ 879
	8192	4096	4165	50842	,,	+ 842
th	2048	512	494	24121	25000	− 879
	4096	1024	1001	24438	,,	− 562
	6144	1536	1548	25212	,,	+ 212
	8192	2048	2028	24755	,,	− 245
2th	8192	1024	982	11987	12500	− 513
3th	,,	512	480	5859	6250	− 391
4th	,,	256	266	3248	3125	+ 123
5th	,,	128	132	1611	1563	+ 48
6th	,,	64	71	867	781	+ 86
7th	,,	32	36	439	390	+ 49
8th	,,	16	17	208	195	+ 13
9th	,,	8	9	110	97	+ 13
10th	,,	4	2	25	49	− 24
11th	,,	2	1	12	24	− 12
12th	,,	1	0	0	12	− 12
13th	,,	$\frac{1}{2}$	1	12	6	+ 6
14th	,,	$\frac{1}{4}$	0	0	3	− 3
15th	,,	$\frac{1}{4}$	2	25	2	+ 23

It is thus seen that the observed corresponds nearer and nearer with the calculated average as the number of trials increases for *h*, and for the three first for *th*, but for *th* it is evident that the number of trials was not nearly enough. After *th* the discrepancies become still greater. The table is then reduced to the case of 8192 trials, which gives the sum of four different sets of 2048 each. It is remarkable that in two of the sets *10th* occurred once, and in one of the sets *11th, 12th, 13th* each occurred once, while in two of the sets *15th* occurred once. The table is arranged by *heads*, each trial ending with an *h*. But the tosses observed might have been arranged by *t*. Then the actual 32 trials at the beginning of this note would appear as *hht, hht, t, hhht, hht, hht, hhhhhht, t, t, ht, hhhht, t, t, t, t, ht, t, t, t, ht, ht, ht, t, t, t, hht, t, hhh* ... giving only 27 trials ending in *t* and part of another trial, and the trials would have had to be continued. It is best to use such a number of trials as 32, 64, 128, &c., because these numbers divide so often by 2 exactly. Hence $2048 = 2^{11}$, $8192 = 2^{13}$, were selected for the trials in the table. A few examples of this kind actually worked out from a child's own experience will do more to convince him of the reality of a law of averages than a month's talk or a year's reading. The *fluctuations* "found" above and below the "expected" average will also show him the meaning of "leaving a margin."

choice with regard to coming into existence or ceasing to exist; we have no choice as to living by air and food; we have no choice as to breathing air or water, flying or walking, having two legs or four, and so on; but we never feel restraint respecting these matters. A prisoner who submits to be taken to prison feels restraint, because he has the choice of going or of being afflicted for not going, and he has all further choice withheld, and so on. In involuntary agents, restraint consists in the introduction of unusual conditions, which bring the event under the action of additional laws. Thus a ball, when thrown, describes a certain path under the action of gravitation. But if a string had been fastened to it, the motion would have been greatly changed. The string introduces a new set of laws. Properly speaking, *restraint* always *consists in coming within the influence of unusual laws.*

iv. *Cause and effect* are two extremely common and very loosely employed terms. Scientifically they *are* used for *two sets of events which occur in the order fixed by scientific law.* But even scientifically the whole of each set of events is rarely thought of; many being so much a matter of course, that their absence rather than presence would have to be noted. But popularly the one seen and observed event which, like the spark applied to a train of gunpowder, is immediately followed by the change, is called the *cause*, and the change itself the effect.* But, in this very instance, it would be absurd to say that a spark was the cause of the ruin of a fort. Not only the gunpowder, but a certain collocation of the gunpowder with respect to the fort, and a certain constitution and dryness of the powder, were also required, and formed part of the scientific cause. Hence *cause* and *effect* are words rather to be avoided,† and the teacher must be careful that they are not used vaguely.

* See Method of Differences, Art. 82.

† Prof. Clifford (p. 509), referring to the common phrase, "every effect has a cause," asks, "What do we mean by this?" and proceeds to say, "In asking this question we have entered upon an appalling task. The word represented by *cause* has sixty-four meanings in Plato, and forty-eight in Aristotle. These were men who liked to know as near as might be what they meant; but how many meanings it has had in the writings of the myriads of people who have not tried to know what they meant by it, will, I hope, never be counted.... I shall evade the difficulty by telling you Mr. Grote's opinion. You come to a scarecrow, and ask, What is the cause of this? You find that a man had made it to frighten the birds. You go away, and say to yourself, 'Everything resembles this scarecrow: everything has a purpose.' And from that day the word *cause* for you means what Aristotle meant by *final cause*. Or you go into a hairdresser's shop, and wonder what turns the wheel to which the rotary brush is attached. On investigating other parts of the premises, you find a man working away at a handle. Then you go away and say, 'Everything is like that wheel. If I investigated enough, I should always find a man at a handle.' And the man at the handle, or whatever corresponds to him, is from henceforth known to you as 'cause,' and so generally. When you have made out any sequence of events to your entire satisfaction, so that you know all about it, the laws involved being so familiar that you seem to see how the beginning must have been followed by the end; then you apply that as a simile to any other events whatever, and your idea of cause is determined by it. Only when a case arises, as it always must, to which the simile will not apply, you do not confess to yourself that it was only a simile, and need not apply to everything, but you say, 'The cause of that event is a mystery which must remain for ever unknown to me.' On equally just grounds the nervous system of my umbrella is a mystery which must for ever remain unknown to me. My umbrella has no nervous system; and the event to which your simile did not apply, has no cause, in your sense of the word. When we say that every effect has a cause, we mean that every event is connected with something in a way that might make somebody call that the cause of it. But I, at least, have never yet seen any single meaning of the word that could be fairly applied to the *whole* order of nature."

The *invariable unconditional order* of events is a much more important conception, and one to which the former should be always reduced. The invariability and the unconditionality are, however, often difficult to establish, and we are not entitled, when we see a certain order always existing in all the observations we have made on two events, to suppose that that order *is* invariable *and* unconditional. The order of day and night is one of the most invariable observed; we know, however, that it is conditional on the positions of the sun and earth, and the rotation of the earth. This is the distinction of *post hoc* (after this) and *propter hoc* (because of this).* We cannot declare the latter unless we feel that there is sufficient evidence of invariability *and* unconditionality; this it is the province of scientific induction to investigate; the mere succession is patent from a single observation.

v. When the invariability and unconditionality cannot be thoroughly established, they may be found to hold within certain limits, so that, the limiting condition being observed, others may be left out of consideration. This gives rise to experimental or *empirical laws*, which are of the utmost value as provisional rules,

* Many writers are of a different opinion; but so far as science is concerned this difference is inoperative. Dr. Carpenter (*ibid.*) says:—"As Sir John Herschel most truly remarked, the universal consciousness of mankind is as much in accord in regard to the existence of a real and intimate connection between cause and effect, as it is to the existence of an external world; and that consciousness arises to every one out of his own sense of personal exertion in the origination of changes by his own individual agency." That sense of personal exertion arises, we may be said to know, from the corresponding physiological changes within him. It therefore only removes the invariable unconditional relations one step further off. But let us go back to the antecedent of these changes, to the *causa causarum*, and we at once transgress the limits of verifiable science, and hence sail beyond our present purpose. It is widely different to say that the invariable unconditional *verifiable* relation is the only one to be considered, and to say that *that* relation is *not* itself a consequent of an unverifiable (or even possibly at some future period verifiable) antecedent. The latter statement is thoroughly unscientific. Dr. Carpenter considers *force*, as deduced from considerations of personal exertion, to be the invariable antecedent of causation, and says, "whilst no 'law' which is simply *a generalization of phenomena* can be considered as having any *coercive action*,"—'law' is merely a statement of invariability and unconditionality without reference to *coercion*,—"we may assign that value," viz., the possession of coercive action, "to laws which express *the universal conditions of the action of a force* whose existence we learn from the testimony of our

and as grouping phenomena for subsequent inductions.* Thus the effect of friction is taken to be exactly proportional to the pressure, but this strict proportion ceases when the weight is large. Part of a beam of sunlight will be reflected from an unsilvered glass surface, according to the usual law of reflection, and part will pass through. Attempt to reflect both portions from another piece of unsilvered glass. The amount of the second reflections will differ materially according to the angles and planes of both reflections, and it is possible so to arrange the positions of the glasses that either the light reflected or transmitted by the first glass will either not be transmitted or else not reflected respectively by the second. The law of reflection and transmission therefore becomes conditional. The investigation of this condition produced within the present century the whole theory of polarized light. Such an experiment requires next to no apparatus, and will amuse as well as instruct the children. But the teacher must have carefully tried the experiment himself before attempting to show it or to lead a child to make it. Those teachers who think they can perform an experiment in public, which they have not easily succeeded in performing in private, have seriously misconceived their vocation.

vi. The *reason why* is constantly asked, and has two meanings requiring discrimination. As respects the acts of a known voluntary agent, we suppose an acquaintance with laws of nature, and with the method of putting himself or his actions within the influence of certain laws; when therefore he does so, we presume that he desired to bring about the consequent effect. The answer to, Why did you do so? is consequently expected to be, Because I wished to get so-and-so. That is, the *reason why* expresses a *motive*, a desire, an intention, a purpose, an object. A large induction shows that human actions are regulated by such motives, and

consciousness"—that is, from ourselves coming consciously under the relation it expresses. "The assurance we feel that the attraction of gravitation *must* act under all circumstances according to those simple laws which arise immediately out of our dynamical conception of it, is of a very different order from that which we have in regard (for example) to the laws of chemical attraction, which are as yet only generalizations of phenomena." The distinction is chiefly one of degree. It is the infinite number of verifications to which science has accurately, and common experience roughly, subjected gravitation, as compared with those to which chemical theories (constantly changing since Priestley's discovery of oxygen) can be subjected, which gives the real balance in favour of the former. Force is *only* measurable by motion: as Dr. Carpenter says (*ibid.*) "the mechanical philosophy of the present day tends more and more to express itself in terms of *motion* rather than in terms of *force*." In English books on mechanics the "force of gravity" is always expressed by the number 32·2, representing the number of feet which a body would describe horizontally on a frictionless surface *in vacuo* during one mean second of time, if it were to move uniformly with the velocity it had acquired after falling vertically *in vacuo* for one mean second of time previously. Any other mode of measuring this force is almost "inconceivable" at the present day; but how could it have ever been discovered or used "from the testimony of our consciousness"?

hence we naturally inquire into the motives of any human action. Frequently, however, we find that the reason why a human being produced a certain result was not that he intended it, but that, intending something else, and ignorant of all the conditions necessary for attaining it, he ended in doing what he did not in the least intend. When the conditions were such as he could not anticipate, their occurrence is given as an explanation of his failure. Thus boys questioned on some school offence, allege such and such unexpected conditions as an *excuse*. "Why were you late?" "I fell as I was running to school." "Why did you not write the exercise?" "I hurt my hand."

vii. But when we get beyond human or living actions, it is not correct to assume intentions. Such an assumption has however often been made supernaturalistically, and leads to *final causes*, the intention of a supernatural being, or of an abstract being, nature herself. But these are excluded from induction proper by the total absence of verifiability (art. 74). Suppose the question were, "Why did such a boy fall ill and die?" No such answer as, "Because it was the will of God," can be admitted in induction, for it is quite evident that it cannot be verified. The only answers would be such as a physician would give, describing the constitution of the boy, the conditions to which he was exposed, the growth and treatment of the disease, and so on. Even these answers are only partially verifiable by a registration of similar cases, and the whole facts on which repose the science of life. How can the *reason why* be given scientifically? *By showing the known laws which include the particular case.* Nothing more should be attempted, and very frequently even this cannot be done.† It is also evident that we must soon come to the end of

* Properly speaking, all the laws that we discover are empirical, limited by our powers of verification. But relatively to us they are perfect when they give results which are as exact as, or are more exact than, our means of verification. Prof. Clifford shows that such considerations apply even to the laws of geometry, as, for example, to the conclusion that the three angles of a plane triangle are exactly equal to two right angles. (*Ib.*, p. 504.)

† Prof. Clifford, alluding to an illustration, says, "The explanation describes the unknown and unfamiliar as made up of the known and familiar, and this, it seems to me, is the true meaning of explanation" (p. 508, col. 1). "By known and familiar, I mean that which we know how to deal with, either by action in the ordinary sense, or by active thought" (p. 508, col. 2). "That a process may be capable of explanation, it must break up into simpler constituents which are already familiar to us. Now, first, this process may itself be simple, and not break up; and secondly, it may break up into elements which are as unfamiliar and impracticable as the original process . . . . It is no explanation to say that a body falls because of gravitation. . . . . . This attraction of two particles must always, I think, be less familiar than the original falling body, however early the children of the future begin to read their Newton. Can the attraction itself be explained? . . . . The attraction may be an ultimate simple fact; or it may be made up of simpler facts utterly unlike anything that we know at present; and in either of these cases there is no explanation. We have no right to conclude, then, that the order of events is always

our chain, and get to laws which we are unable to include in any higher, and then all "reasons why" are at an end." "Why was this plate smashed?" "A stone fell upon it." "Why did the stone fall?" "The wind blew it from the top of the wall." "Why did the wind blow it?" "There was a sudden powerful gust in such a direction." "But why sudden, why sufficiently powerful, why in this direction?" "I don't know."—"Why was that stone on the top of the wall?" "The builders placed it there as a coping." "Why was it not tight?" "The mortar had ceased to hold it?" "Why?" "I don't know."—"Why did the stone fall on the plate, and not beside it?" "Because the plate was placed in a certain position, and the wind blew with a certain force." "Why?" "I don't know."—"Would a little stone have smashed the plate?" "Probably not." "Why?" "Because it would not have acquired sufficient force in falling." "Why?" "This can be calculated from its weight, the height from which it fell, the law of gravitation, and the cohesion of the parts of the plate." "Why was the weight or the height such? Why does the force of gravitation exist? Why do the parts of a plate so cohere?" "I don't know."—And so on. It is often useful to follow up such a train of questions (here purposely much curtailed) and then finally to show that all the reason given was the inclusion of facts within laws, and these within more general laws, so that the result was simply showing the *manner*, not the *motive* or purpose, and that, using *why* in the human sense, we may sum up the result by saying, *inductive science seeks to know the how, and not the why.**

capable of being explained" (p. 509, c. 1). At the same time, we have no right to conclude that any given order of events will remain for ever inexplicable. As Prof. Clifford says in reference to another question, "It seems to me that we do not know, and that the recognition of our ignorance is the surest way to get rid of it" (*ib.* p. 505, c. 1).

* In the words of Comte, inductive science "écarte comme radicalement inaccessible et profondément oiseuse, toute recherche sur les *causes* proprement dites, premières ou finales, des événements quelconques. Dans ses conceptions théoriques, elle explique toujours *comment* et jamais *pourquoi*. Mais, quand elle indique les moyens de diriger notre activité, elle fait, au contraire, prévaloir constamment la considération du but." *Cath. Pos.* p. 13. Dr. Carpenter concludes his presidential address thus:—"The science of modern times . . . . fixing its attention exclusively on the *order* of Nature," that is, of phenomena—"has separated itself wholly from theology, whose function it is to seek after its *cause*. In this, science is fully justified alike by the entire independence of its objects, and by the historical fact that it has been continually hampered and impeded in its search for the truth as it is in nature"—for the discovery of invariable unconditional relations, giving the power of prediction and consequent verification—"by the restraints which theologians have attempted to impose on its inquiries. But when science, passing beyond its own limits, presumes to take the place of theology, and sets up its own conception of the *order* of nature as a sufficient account of its *cause*,"—I do not know any man of 'science' who has done so; the words seem aimed at Comte, to whom the preceding quotation shows that they are inapplicable; the whole statement should have been worded hypothetically,—"it is invading

viii. One other word, *nature*, remains to be considered. It has been greatly abused, and is often used euphemistically for a supernatural power. At other times it means the universe itself, or its constitution, or the laws which any object obeys; sometimes it is even used for all the universe except man, or except animals. It must consequently be employed with caution. If we talk of a "natural" spontaneous action, we mean an action according to the laws which we usually see affecting the object, so that there is no *restraint*, in the sense already explained. When we "naturally" conclude that so and so is the case, we draw our conclusions from such knowledge as we recollect at the moment, without investigation. Policemen see a man stagger from a public-house, and fall down. They "naturally" conclude that he is drunk, and lock him up. But in several instances of the kind, the man has been found dead the next morning, having fallen in a fit. "Naturally" often means "hastily, negligently, ignorantly."

a province of thought to which it has no claim, and not unreasonably provokes the hostility of those who ought to be its best friends;" and would be so if they had had sufficient education in those merest elements of science to understand what science does seek to attain, and not to attribute to science aims utterly alien to any possible conception of science admissible in these days. "For whilst the deep-seated instincts of humanity, and the profoundest researches of philosophy, alike point to mind as the one and only source of power"—in man, to assume the same, or anything approaching to the same superhumanly, is to emulate Phaethon—transnature is degraded by reducing it to cis-nature—"it is the high prerogative of science to demonstrate the *unity* of the power which is operating through the limitless extent and variety of the universe,"—how if the power itself is beyond scientific ken? really, science strives to demonstrate the harmonious character of the invariable unconditional relations; it can't do more, and it can only at present forefeel, scarcely state, far from demonstrate, even this—"and to trace its *continuity*"—continuance of the invariability—"through the vast series of ages that have been occupied in its evolution." The last words appear to be a mistake, for "in the evolution of the universe," see the conclusion of this note. It is curious that Dr. Carpenter should have been betrayed into thus committing the very mistake that he deprecates,—transplanting religion into science. That he has done so, and that it was one of the very objects of his address to do so, Dr. Carpenter admits in a letter subsequently published in the *Echo* (the date of which I am unable to give), where he says: "I expressed the opinion that science points to (though at present I should be far from saying that it demonstrates) the origination of all power in mind; and this is the only point in my whole address which has any direct theological bearing. When metaphysicians, shaking off the bugbear of materialism,"—which is generally the application of lower laws to higher phenomena, and is *at best* a doubtful analogy,—"will honestly and courageously study the phenomena of the mind of man in their relation to those of his body, I believe that they will find in that relation their best arguments for the presence of Infinite mind in universal Nature." This lies beyond the purpose of a purely scientific investigation. It is important that intellectual verifiable science, and emotional unverifiable religion, should be sharply distinguished. The following quotations from the same letter throw further light on Dr. Carpenter's notion of 'coercive' laws. "My object"—in the Brighton address—"was to show—(1) That what we call 'laws of nature' are simply

80. The province of scientific induction then is to discover *laws*, which take the form of general assertions respecting such matters as are appreciable by the senses, and are hence called *appearances* (Latin), or *phenomena* (Greek). These appearances are, however, generally supposed to refer to some *underlying beings* (English) or *substances* (Latin) which merely exist in our *thought*, and are hence called *noumena* (Greek). This distinction is, of course, not one which can be proposed to children in *terms*, but in the course of object lessons it can be easily shown that all we *know* of *things* are their *appearances* to our senses; that in fact we apply the *names* as merely a short way of *tying together* (English) or *colligating* (Latin) these appearances, and that we know nothing about the things *themselves* (*die Dinge an sich*, as the German philosophers word it). The process of such observation gives first an *impression* on our senses, which is regarded as an *appearance*, and then we have the *recollection of this appearance*, often called an *idea* (Greek) or *notion* (Latin). It is with these ideas or notions that we think, but *these internal images are always less precise and definite than the appearances as derived from external impressions*,

our own expressions of the orderly sequence which we discern in the phenomena of the universe; and that as all the history of science shows how erroneous these have been in the past, so we have no right to assume our present conceptions of that sequence to be either universally or necessarily true. (2) That these so-called 'laws' are of two kinds—one set being merely generalisations of phenomena, of which Kepler's 'Laws of Planetary Motion,' or the 'Laws of Chemical Combination,' are examples; while another set express the conditions of the action of a force, of which the existence is, or may be, made known to us by the direct and immediate evidence of our own conscience—our cognition of matter being indirectly formed through the medium of force. (3) That 'laws' of the first kind (which we may for convenience term phenomenal) do not really explain or account for anything whatever. Nothing is more common than to hear scientific men speaking of such laws as 'governing phenomena,' or of a phenomenon being 'explained' when it is found to be consistent with some one of such laws; though the fact is, that the law is a law, merely because it is a generalised expression of a certain group of phenomena; and to say that any new phenomenon is 'explained' by its being shown to be in conformity to a 'law,' is merely to say (as Professor Clifford well put in his lecture) that a thing previously unknown is 'explained' by showing it to be like something previously known." This is hardly a proper interpretation of Prof. Clifford's words, given in the preceding note. "(4) That on the other hand, every 'law' of the second kind (which we may distinguish as dynamical) is based on the fundamental conception of a force or power; so that if the existence of such a force (as that of gravity or electricity) be admitted, and the conditions of its action can be accurately stated, then the law which expresses them may be said to 'govern' the phenomenon; and any phenomenon which can be shown to be necessarily deducible from it may be said to be 'explained' so far as science can explain it. But the utmost that science can positively do, as I stated towards the conclusion of my address, is to demonstrate the unity of the power of which the phenomena of nature are the diversified manifestations, and to trace the continuity of its action through the vast series of ages that have been occupied in the evolution of the universe." The text and preceding remarks will enable the reader to appreciate these explanations.

and hence they require to be constantly refreshed by recurrence to these actual appearances, that is, to *observation*, either on the events which occur to us without any planning on our part (*natural observation*), or on events artifically arranged (*experimental observation*). Scientifically we must be considered to have *no* ideas or notions which have not been thus derived. *Conceptions* are properly internal constructions of ideas thus derived, and must hence be always subordinated to external materials. When they are not, we say, in very evident cases, that a thinker is *mad*, or subject to illusions. In other cases, we soften the term, and talk of prejudice, or eccentricity, or bias. In all cases we acknowledge untrustworthiness.* Whom are we then to trust? It is clear to anyone who has attempted to make any inductions, that his own individual range of thought is very limited. When he looks upon the lives, the many generations of lives, exhausted over obtaining a few results, he sees also the absolute necessity of trusting, and the difficulty of trusting even himself. *The test of any person's authority is simply the degree of verification which his results have received.* But even this amount of verification we have constantly to take on evidence. Our trust, our absolute *faith*—for it comes to that—can only be founded on our own personal knowledge of the *kind* of processes by which results are obtained and verified. Hence the importance of an early familiarisation of the minds of children with the great processes of induction in matters which they *can* appreciate. But we must recollect that to *talk* to them of things they have not seen, or felt, or sensuously appreciated, is only to confuse their minds and waste our own strength. All induction is founded on facts derived from natural or experimental observation, and one business of the teacher in *all* he teaches is to make that observation as thorough and precise as possible.

* As Dr. Carpenter (*ibid.*) says: "The scientific conception of nature . . . is a *representation framed by the mind itself* out of the materials supplied by the impressions which external objects make upon the senses; so that to each man of science *nature is what he individually believes her to be*." Whence the importance of the verification of scientific theories by others than their proposer. "It may be said," he adds, "is not this view of the material universe open to the imputation that it is 'evolved out of the depths of our own consciousness,'—a projection of our own intellect into what surrounds us—an *ideal* rather than a *real* world? If all we know of matter be an 'intellectual conception,' how are we to distinguish this from such as we form in our dreams? . . . Here our 'common sense' comes to the rescue. . . . Every healthy mind is conscious of the difference between its waking and its dreaming experiences . . . . and . . . finding its own experiences of its waking state not only self-consistent, but consistent with the experiences of others, accepts them as the basis of its beliefs, in preference even to the most vivid recollection of its dreams. The lunatic . . . . is so 'possessed' by the conception framed by a disordered intellect, that he *does* project it out of himself into his surroundings; his refusal to admit the corrective teaching of common sense, being the very essence of his malady. And there are not a few persons abroad in the world, who equally resist the teachings of common sense, whenever they run counter to their own preconceptions; and who may be regarded as—in so far—affected with what I once heard Mr. Carlyle pithily characterize as a 'diluted insanity.'"

Hence the importance of an early introduction of some science of natural observation, as *botany*, which, at least in country places, and partly even in towns, can be made such a valuable and interesting study, as has been shown in several recent works.* And this, or some similar science of observation, becomes the best vehicle for the introduction of strict views respecting scientific names, terminology, and classification, to which considerable extension can be given when pupils have gone through a course of deductive logic, but which can be brought home to the feelings of pupils who have never even heard of logic. The great *principle of natural classification*† *by the increasing or decreasing generality of the appearances exhibited, or the laws obeyed, by the objects or events classed*, can readily be shown even without taking up a systematic study. Children are by natural observation sufficiently

* La Maoût's Leçons Elémentaires de Botanique (16 francs) recommended by Mr. J. M. Wilson of Rugby, obtainable at Hachette's, King William Street, Strand. Miss Youmans' First Book of Botany (King, 65 Cornhill). Miss Youmans' Culture of Observing Powers of Children, edited by Mr. J. Payne (King, 2*s.* 6*d.*), is published separately. Dr. Master's Botany for Beginners (3*s.* 6*d.*), 100 illustrations (Bradbury and Evans).

† The classification of deductive logic is *artificial*, proceeding by the presence and absence of unmistakable marks, as the presence or absence of certain letters in a word (arts. 7 to 11). But in *natural* classification this sharp determination of marks no longer exists. Concretely the marks relied upon shade by degrees into each other, till it becomes difficult to determine whether a given object can be rightly said to have or not to have the assigned mark (quality or property). The Linnæan (*artificial*) and the Jussieuan (*natural*) classifications of plants are very useful in familiarising the mind with this difference. But even in broader distinctions, as between plants and animals, it will be as well to take an opportunity of showing how difficult it is to say whether a given organism (for example, a *Diatom*) is a plant or an animal, and hence to lay down unmistakable characteristics which should determine that a man and a zoophyte are animals, and a Sensitive Plant or a Venus's Fly-trap (*Dionaca muscipula*) are not animals. Much harm has been done by confusing artificial and natural distinctions, and by forgetting that language is founded upon loose observation of natural differences which are treated as artificial and strict. One of the great sources of difficulty in judging of the permanence of species, or natural character of genera, arises from this very looseness. Hence, again, in all cases, the danger of pushing home mere verbal distinctions. Observe how loosely we use such words as hot, cold, black, white, full, empty. One would think these must be absolute terms. But we have hotter and colder, blacker and whiter, fuller and emptier! To draw the line between hot and cold, black and white, full and empty, becomes really difficult. Don't blame a boy, as I have heard boys blamed, for saying that one cup of tea is "fuller" than another, or that a third is "almost quite empty." They are merely endeavouring to give a precise impress to words long worn smooth by use. But the teacher may well "improve the occasion" from time to time, and lead a boy to think more accurately, when he catches them out in such tricks. A few laughing words will often be of much greater service on such an occasion than an hour's serious lecture. There is a chance of the joke being remembered through life. The hour's lecture, like other bores, will be certainly forgotten. See also the quotation from Prof. Clifford, art. 74, last note, p. 66.

acquainted with a number of animals, vegetables, and inorganic substances, to render the classifications of animate and inanimate quite distinct, and to make the separation of animal and plant life easy. They will learn to appreciate the great laws relating to all the three sets, and how these are affected by plant life, and still more by animal life. But these special considerations belong rather to science teaching, than to the few general remarks to which I am forced to limit myself.

81. The so-called *canons of induction* need not occupy us much, for school life does not afford many instances by which they can be exemplified. They are (as already stated) merely elaborations of the first principle of forming the simplest possible supposition (art. 76), and serve to show how that supposition may be corrected, and how it must almost always be considered as provisional. They are, as you know, the methods of *agreement*, *difference*, *concomitant variations* and *residues*, which may be used separately or jointly, and are chiefly applicable for experimental research. The object is in every case to determine between which two sets of events there exists an invariable unconditional relation. One of the first discoveries made is, that one set of events may be preceded, and apparently invariably and unconditionally preceded, by totally different sets of events. Thus, a man's death is the consequence of certain diseases, certain poisons, certain wounds, certain constrictions, &c., so that, given a dead man, to find how he came by his death, is often a difficult, sometimes an impossible problem to resolve, as shown by coroner's inquests. In making deductions from inductions, we are obliged, from our present ignorance of the logic of sequence, to reduce the problem to one of consistency of assertions, thus:—Let the assertion,

"the events A come first," be X",
"the events B come first," be Y",
"the events M come second" be Z."

Then †X.*z*, †Y.*z* represent the assertions, "if the events A (or B) come first, the events M come second," (art. 59, b.) On developing the series of complexes (art. 54) we obtain

X.Y.Z   X.*y*.Z   *x*.Y.Z   *x*.*y*.Z
†X.Y.*z*   †X.*y*.*z*   †*x*.Y.*z*   *x*.*y*.*z*.

Consequently the occurrence of the events M as second are consistent with the occurrence of either or both or *neither* of the events A and B as first, and the only conclusion which can be drawn is, that if the events M do *not* occur, neither of the sets of events A or B can have preceded. This is the ambiguity which arises from what is called the *plurality of causes*, and which besets us in endeavouring to find the *cause* from the *effect*.*

* Dr. Carpenter (*ibid.*) gives as an example of this Mr. Lockyer's speaking "as confidently of this sun's chromosphere of incandescent hydrogen and of the local outbursts which cause it to send forth projections tens of thousands of miles high as if he had been able to capture a flask of this gas, and had generated water by causing it to unite with oxygen"—hydrogen being the only known gas which will do so, whereas hydrogen has not been proved to be the only extra-mundane gas which when incandescent

But here a new point has come to light. Both sets of events A and B might come first; that is, they might occur simultaneously before the events M. Do sets of events which concur modify each other or not? Here we come to three laws, which were first only known as laws of motion, but which science has come to extend over the whole of its region. They are extremely extensive inductions, and can be only partially illustrated. The real proof is in the verification of the deductions made from them. Succinctly stated, they are,—

i. The Law of *Persistence.* Every condition of things tends to remain unaltered, and resists external attempts at alteration.

ii. The Law of *Composition.* Every action upon a system or condition of things produces its whole effect, without regard to any simultaneous action.

iii. The Law of *Reaction.* Every action upon a system of things calls up a contrary, and, when properly measured, a precisely equivalent action.

The law of *persistence* is however partly due to an hypothesis by which internal are replaced by external actions; and that of *composition,* to the consideration of so-called generated forces as new forces, bearing in mind the great law, quite recently evolved by men still living, the *conservation of energy,* showing that no really new force is ever generated, but that such apparent new forces are but transformations of others which have superficially disappeared.* These matters can of course only be touched on in passing. It will simply be the teacher's business, in any investigation, to bring back the pupils' thoughts when they wander from these secure paths.

Analogy is not so much an inductive method, as a means of suggesting the direction for inductive investigations. Two sets of events agree in a number of particulars. The first has certain consequences. We conceive that the second will have consequences also agreeing in many particulars with the first. This is merely a simple supposition, founded on induction based on such observations. But observation also shows that the analogy frequently breaks down, and hence we now use it simply as a suggestion. The analogies of sound and light suggested the undulatory

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will produce a certain coloured line on being viewed through the spectroscope, on which Mr. Lockyer's conclusions were based. For the same reason he shows that a new yellow line which has been observed in the spectroscope need not necessarily be due to a new metal helium. In fact, the state of temperature at the sun's surface so far exceeds any on which we can experiment on earth, that the incandescent substances in the sun may affect our spectroscope differently from those on earth.

* Perhaps this conservation of energy may be considered as merely a phase of the law of persistence, when stated as by Dr. Carpenter (*ibid.*), "Energy of *any* kind, whether manifested in the 'molar' motion of masses, or consisting in the 'molecular' motion of atoms, *must* continue under some form or other without abatement or decay." The forms, however, are so different, that it is only by the precisely equivalent mechanical effects that we infer the identity or conservation of the energy. The law of persistence, however, extends to social relations.

theory for the latter; but this has to repose upon its own merits, and the whole nature of wave motion in the latter had to be conceived so differently from that in the former, that the analogy rapidly ceased to be useful. Ordinary language is full of analogies arising from vague metaphors or partial resemblances hazily seized, and the results of "pushing these home" are often sufficient to discredit them. But, in all our formations and applications of inductive laws, we assume that there are events and objects so similar, that if one be substituted for the other, the results could be substituted one for the other. This *interchangeability* is distinct from *analogy,* although it probably suggested it, because the interchangeability is only partial, not total. Thus, in respect of being an object of thought separately conceivable, a mouse is interchangeable with an elephant, and on such grounds rest all the inductions respecting number. In respect of being fluid at ordinary temperatures, water is interchangeable with mercury. By investigations of waves formed in mercury, important conclusions were consequently obtained for waves of water. But here the resemblance was so great, we should hardly call the effects analogous. The resemblance between the government of a family by the father, and of a state by the sovereign, is however merely an analogy, suggesting much, but not giving a ground for transference of conclusions.

82. i. The method of *agreement* is simply this. A great number of different sets of *antecedents* (or events which come first) having been naturally or experimentally observed, in all of which the *same* events A occur, and it being found, on noting all the corresponding different sets of *consequents* (or events which come second,) that the *same* set of events M occur in all of them, it is assumed as the simplest (of course provisional) supposition that there is an invariable, unconditional relation between the two sets of events A and M, so that *whenever* A occurs (and not merely in the cases hitherto observed) we shall look to see M, and whenever M occurs, we may possibly learn that A preceded. Having made this assumption, we reduce it to a formal assertion, and use it as in our previous deductive logic, in connection with other assertions, and then compare, if possible all, but at any rate some of the conclusions with accurate observation. If the two again agree, we feel more confident. But a great degree of hesitation will always attach to this method owing to the plurality of causes (art. 81, p. 85).

ii. The method of *difference* is much more satisfactory. A set of antecedents and consequents being known and observed, suppose that when one of the antecedents is omitted, the consequents also are observed to become different by the omission of one; or when an additional antecedent is introduced, the consequents are likewise changed by the addition of one. In these cases the simplest supposition is that there was an invariable unconditional relation between the omitted antecedent and consequent, and also between the additional antecedent and consequent. But numerous experiments are necessary to force this scientifically home to the mind, as it is evident some intermediate links may have been

overlooked, or some other additions or omissions unobserved. It has however given rise to the popular use of the word 'cause' (art. 79, iv.)*

iii. The method of *concomitant variations* is a common variety of the method of difference. Here no antecedent is entirely removed or added, but one antecedent is varied in intensity, and a corresponding variation of intensity in one consequent is observed. We must guard against expecting these variations to be of the same kind, or that if one takes place in one direction, the other will also constantly take place in one direction; or that the antecedents will alter in the same ratio as the consequents. It is very commonly said that the effect varies as the cause. This is an attempt to give mathematical accuracy of expression to what has been only vaguely observed. As thus expressed, the proposition is far from being generally true. If you spill a glass of water over a person's hand, the effect is very different from letting a small quantity drop upon the hand gradually. This is an excellent experiment for boys. If you diminish the heat, alcohol continually diminishes in volume; water diminishes to a certain extent, and then increases. This should be shown, for the fact is valuable.

iv. The method of *residues* consists simply in deducting from a set of consequents those known, from previous complete inductions, to be the result of a portion of the antecedents, and then making the simplest supposition that the remaining consequents are due to the remaining antecedents.

We might go on to the combination of the different methods, but the only real point of importance in all these methods for our present purpose, is to understand that they are all referable to the one law of forming the simplest supposition, and that the conclusions are therefore all provisional, requiring constant verification. From this we see the necessity of registering all such conclusions

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* Dr. Carpenter says (*ibid.*): "While fully accepting the logical definition of cause as the 'antecedent or concurrence of antecedents on which the effect is invariably and unconditionally consequent,' we can always single out one *dynamical* antecedent—the power that does the work—from the aggregate of *material conditions* under which that power may be distributed and applied." This distinction of the *powerful* and the *powerless* antecedents, is, at least hazardous, and theoretical. It is never obligatory. "No doubt," he continues, "the term cause is very loosely employed in popular phraseology .... but even a very slightly trained intelligence can distinguish the power which acts in each case, from the conditions under which it acts." In the cases of blowing up by gunpowder, a man falling from a ladder, and printing by steam, Dr. Carpenter makes the "efficient causes" "the force locked up in the gunpowder, ... the gravity which was equally pulling him down while he rested upon it [the ladder ....], and the steam engine outside," respectively, and all the rest, "the material conditions constituting (to use the old scholastic term) only 'their formal cause.'" The whole distinction scientifically appreciable, excluding the consideration of these retrograde metaphysical terms, is shown at the close of art. 78, p. 11. See also the quotation from Prof. Clifford respecting the vagueness of the term *cause* in the note † to art. 79, iv., p. 76.

as have been satisfactorily verified, and *this registration takes the form of a treatise upon a particular branch of science*, as distinct from the general investigation of the principles which lead to the necessity of such investigation. *Induction is the father of science, not science itself.* But science cannot be properly appreciated till its parentage is understood.*

83. In the preceding remarks I have endeavoured to indicate the principal points which a teacher must attend to in teaching induction. On account of the great difficulties which attend the preparation for verification in all great inductions, it is not possible to obtain a complete view of the subject without long and arduous study of the principal inductive sciences. Hence treatises upon Inductive Logic drift more or less into a philosophy of the elementary principles of the special sciences, which cannot be even partially understood without something more than an ordinary household acquaintance with them. Thus almost half of Prof. Bain's work on Induction is devoted to the "Logic of the Sciences," and yet each science is treated with almost bald succinctness. Now it is evident that all this is out of the question in schools, and that it is only in advanced schools with very capable masters, that it can be even partially taught, in respect, for example, to geometry, mechanics, chemistry, sociability. But a few great principles can be always inculcated, on innumerable occasions, so that when the child grows up he may find the forest cleared for the subsequent cultivation of science. These are the principles which I have been trying to present to your notice. If a child leaves school with a firm conviction of the *uniformity of nature*, that is, the *invariability and unconditionality of relations*, and a constant habit of making the *simplest suppositions consistent with a representation of the* WHOLE *of the facts known*, he will be a totally different being, intellectually, from the child who has been taught the usual routine, in which no notice of these great

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* Prof. Clifford concluded his admirable lecture as follows (p. 511, c. 2): "By scientific thought we mean the application of past experience to new circumstances, by means of an observed order of events. By saying that this order of events is exact, we mean that it is exact enough to correct experiments by, but we do not mean that it is theoretically or absolutely exact, because we do not know. The process of inference we found to be in itself an assumption of uniformity, and that, as the known exactness of the uniformity became greater, the stringency of the inference increased. By saying that the order of events is reasonable, we do not mean that everything has a purpose, or that everything can be explained, or that everything has a cause; for neither of these is true. But we mean that to every reasonable question," [previously defined (p. 511, c. 2) as one which is asked in terms of ideas justified by previous experience, without itself contradicting that experience,] "there is an intelligible answer, which either we or posterity may know *by the exercise of scientific thought*. .... Scientific thought is not an accompaniment or condition of human progress, but human progress itself. And for this reason the question what its characters are, ... is the question of all questions for the human race."

principles occurs.* And there seems to be no good reason why a child should not attain to such habits and convictions. The material is all there, and it remains with the teacher to employ it. A few occasions have been pointed out as they arise, from which the method I would adopt generally may be inferred. But some additional special observations seem desirable.

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* In the preceding articles I have made much use of the philosophic laws as stated by Auguste Comte. As these laws are not easily accessible in the form he has assigned, it seems expedient to annex them, as far as possible, in the words of his *Politique Positive*, vol. 4, pp. 173—180, published in 1854. As they are there always stated in an oblique form, I have thought it best to alter the construction, and also to disinter them from the body of the paragraphs in which they occur, and arrange them in the order which Comte has rather indicated than actually carried out. The greater part of these laws are generally recognized, and it is only the form which has been given to them that is peculiar to Comte; this form, however, is of considerable importance. Laws 7, 8, 9 are peculiar to Comte's philosophy, and have been vehemently disputed. Laws 10, 11, 12, 13, in their original form, are Kepler's, Galileo's, and Newton's laws of motion, and D'Alembert's principle, respectively. The law of the Conservation of Energy, due to Mayer, Helmholtz and Joule (art. 81), was unknown to Comte, but he would probably have included it under law 10. At the end of each law here referred to, I have added the number of the article in which it is used and explained, and this will dispense with the necessity of any translation. The law is generally printed in italics in the articles cited. Where no article is referred to, the law has not been used.

"Principes Universels sur le concours desquels doit reposer la systématisation finale du dogme positif. Les quinze lois universelles de la philosophie première, reparties entre trois groupes, dont les deux derniers, doubles chacun du premier, se décomposent respectivement en deux séries égales.

I. Le premier groupe, non moins relatif à la constitution intérieure de nos spéculations qu'à leur destination extérieure.

1. Principe fondamental. Formez l'hypothèse la plus simple que comporte l'ensemble des documents à représenter. (Art. 76, p. 68.)

2. Reconnaissez l'immuabilité des lois quelconques, qui régissent les êtres d'après les événements, quoique l'ordre abstrait permette seul de les apprécier. (Art. 78, p. 70.)

3. Les modifications quelconques de l'ordre universel se trouvent bornées à l'intensité des phénomènes, dont l'arrangement demeure inaltérable. (Art. 78, p. 71.)

II. Le second groupe, directement relatif à l'entendement, se décompose en deux, statique et dynamique.

a. Statique. L'ordre consiste dans l'établissement de l'unité.

4. Subordonnez les constructions subjectives aux matériaux objectifs. (Art. 80, p. 83.)

5. Les images intérieures sont moins vives et moins nettes que les impressions extérieures. (Art. 80, p. 82.)

6. Que l'image normale aie la prépondérance sur celles que l'agitation cérébrale fait simultanément surgir.

b. Dynamique. Le progrès consiste dans le développement de l'ordre.

7. Chaque entendement présente une succession de trois états, fictif, abstrait et positif, envers des conceptions quelconques, mais avec une vitesse proportionnée à la généralité des phénomènes correspondants.

84. Deductive logic can be taught by a continual recurrence to one single class of instances, the occurrence of letters in words. Inductive logic cannot be taught without recurrence to a great variety of classes of instances, as the great variety alone can give sufficient sanction to its conclusions. Deductive logic requires a very close attention to a large number of particulars, which involves a great strain on the mind unless assisted by something approaching to mathematical severity of notation, while the very abstractness of that notation makes it formidable to many minds. Inductive logic, in its widest and simplest principles, can be studied at first without the same attention to minute details, without any approach to a mathematical notation, and by a continual recurrence to facts actually present to the eye or mind, which arouse and fix attention by their concrete character. It is quite true that we cannot proceed even a moderate distance in strict induction, without close deduction (art. 72); but in the familiar instances of daily life, the deductions are usually so simple that they may be assumed as self-evident. The habit of making them by aid of the simplest supposition principle (which the teacher will take care is properly applied) will be an excellent introduction to that stricter form already explained, which is, after all, merely an application of the same principle. Hence we must not wait for instruction in deductive logic before we proceed to induction, and we must therefore not begin by teaching induction systematically. The object is to lead the young mind by mere work to appreciate some of the great principles upon which

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8. On reconnait une progression analogue pour l'activité, d'abord conquérante, puis défensive, enfin industrielle.

9. On étend aussi la même marche à la sociabilité, d'abord domestique, puis civique, enfin universelle.

III. Le troisième groupe, où domine l'objectivité, se divise en deux séries, celle des plus objectives, et celle des plus subjectives.

a. Les plus objectives ne furent d'abord appréciées qu'envers les phénomènes mathématiques.

10. Tout état, statique ou dynamique, tend à persister spontanément, sans aucune altération, en résistant aux perturbations extérieures. (Art. 81, p. 26.)

11. Un système quelconque a l'aptitude à maintenir sa constitution, active ou passive, quand ses éléments éprouvent des mutations simultanées, pourvu qu'elles soient exactement communes. (Art. 81, p. 26.)

12. Il existe partout une équivalence nécessaire entre la réaction et l'action, si leur intensité se trouve mesurée conformément à la nature de chaque conflit. (Art. 81, p. 26.)

b. Les plus subjectives, où l'origine mathématique devient moins appréciable.

13. Subordonnez partout la théorie du mouvement à celle de l'existence, en concevant tout progrès comme le développement de l'ordre correspondant, dont les conditions quelconques régissent les mutations qui constituent l'évolution.

14. Le classement positif s'effectue toujours d'après la généralité croissante ou décroissante, tant subjective qu'objective. (Art. 80, p. 24.)

15. Subordonnez tout intermédiare aux deux extrêmes dont il opère la liaison."

inductions are made, but not itself to make large or accurate inductions. The following are merely suggestions, which an intelligent teacher will readily seize, and which indeed do not in themselves present any novelty.

85. i. The first care must be to put a child into the proper condition of mind to make or appreciate inductions. Hence *accurate observation* must be cultivated. The children must see things and describe them. Enter a room, remain a short time, retire and tell, or better still write down, the names of all the objects you can recollect, (this was one of the exercises of the celebrated French conjurer, Robert Houdin). The room may be exchanged for a shop, and entering for passing. Take a walk, name all the roads turning to the right or left, the bridges, rivers, streets, isolated houses. Pass a house, describe the number of storeys, the nature of the roof, the number and situation, or any peculiarity in the shape of the windows. Describe a person seen, height, sex, age (child, adult, old), colour of hair and eyes, complexion, hands, boots or shoes, dress (especially the colours of its parts, or any peculiarity in its shape). In a walk mention the number of persons seen, and divide them into male and female, and subdivide according to age; also divide first by age, and then subdivide by sex; also mention number of quadrupeds, and number of each kind of quadruped observed. Describe each child in the school (if not large) or in the class, or class-room. Such descriptions and statistics should be reduced to writing, and stated in a tabular form. The above are merely specimens to guide the teacher. Occasions for such observations arise constantly in the course of instruction, in languages, in arithmetic, in geometry, in lessons on objects, especially when concerning minerals, crystals, shells, flowers, animals. *Extreme* accuracy should be always appreciated. The natural powers of observation, however, differ materially. Beware of thinking a quick observer clever, or a slow observer dull. A teacher can hardly make a greater mistake.

ii. Next take *invariable sequences of common occurrence*, with a view of making the *fact* of the existence of such sequences familiar. The rising of the sun and all heavenly bodies in the east, their passing to the south, and setting in the west. Light following sunrise, and disappearing at sunset. Power of vision following lighting a candle in the dark. Falling of a stone unsupported; weariness resulting from supporting a stone in the hand; weariness from any continued exertion. Sleepiness following wakefulness. Wakefulness following excitement. Pain following burns or blows. Hunger and thirst following much exertion, or long abstention from eating and drinking. Heat from approaching fire, or from exercise. Solution of salt and sugar, and non-solution of stones in water. Sinking of stones and non-sinking of cork placed on surface of water. Children growing taller as they grow older; adults not altering in height. Adults growing weak as they grow old. Death following life with or without disease, in vegetables, domestic animals, and man. Budding in spring, fall of leaf in autumn. Meat rendered fit to eat by cooking. Improvement in sports and work by practice. And so on, and so on, the great point being by extremely numerous examples, sure to be familiar to children, to prepare them for the acceptance of the general principle of invariability throughout nature, inanimate and animate, involuntary and voluntary, individual and social.

iii. *Accidents of common occurrence* should also be noted, and it should be shown that in several cases an invariability was unexpectedly brought into action. A boy runs, trips, falls, cuts his hand; here the accident was his tripping, that is, his having so wrongly measured his distance as to strike the stone; this was "avoidable," the rest "unavoidable," an important distinction. Two boys running round a corner in opposite directions collide with pain; here there was ignorance; see whether it was altogether unavoidable. A boy writing is startled by a sudden noise and blots his book; which is the accident—the noise, the start, or the blot? After many such notions of accident, proceed to others, as the state of the weather on certain days of the month, at certain changes of the moon, &c. As these changes are badly remembered, make boys record them, as well as they can. It is very important to upset the prophecies of weather almanacs, especially in agricultural districts. Show, however, that weather obeys the law of averages, so that we *expect* a certain kind of weather in certain months. See the former exercises (art. 79, ii. p. 73.)

iv. *Simple inductions* may now be introduced. The first kind which I would suggest are not the most simple in themselves, but are most simple to the children, because they have probably been in the habit of making them. They consist of investigations into what has happened, inquiries into circumstances of school and social life. Thus, "Has John been here?" Some children may have seen something which John usually brings, a book, a hat, a parcel, and hence conclude he has been here. This should be followed up. "It is a very good simple supposition, but could not the things have come otherwise? Are they *always* brought by John, and no one else?" We have here the plurality of causes exemplified. Again, a dispute arises in a game; the evidence on each side is taken, the importance of accurate observation appears in contradictions where good faith is assumed; the necessity of deliberation is shown; inferences will have been probably substituted for the statement of direct observation; the differences between eyesight and hearsay will be displayed; the impossibility of decision, that is, of making a safe induction, may become apparent, or hasty decisions may be shown to be false. An event happens in a town, there is a trial reported, a question of identity or alibi arises; the evidence must be scrutinized. And so on.

v. The next class of inductions are really more simple, but require more careful and exact treatment, and hence have to be taken last. Two stones are let fall from the same height, and reach the ground at the same time. Does this depend on their weights, actual or relative, on the particular height? Vary both. Very light weights, pieces of paper, or feathers, take a longer time in falling. Show that this time can be increased by creating a wind: show that if, by attaching a little weight, the paper be made to fall sideways in calm air, it more nearly falls in the same time as a heavy stone. Show that if upon a flat weight (as a penny) a smaller piece of paper be laid, both the paper and

weight will fall in the same time. Make simple suppositions with respect to time of falling being independent of weight but dependent on air. Experiment on resistance of air, generally, by moving hand or flat body, full face or sideways. Similarly in water. Experiment with kites and oars. Experiment with throwing bodies having motions of rotation. Experiment with throwing and dropping in water. Two weights slung over a pulley, give a rough Atwood's machine, and lead to many valuable results, as to rate of falling in equal times, as to time of falling depending on sum and difference of weights. Make pendulum of thread and bullet of stone, find law of variation of its length as compared with its time of vibration. Find length of seconds pendulum, by counting oscillations—swings is the child's word—and difference of rates of different pendulums by observing their coalescences. Apply to clocks, beating time in music, &c. Reflection of fives balls, carefully observed and measured, giving average angle of reflection equal to average angle of incidence, and leading to the induction that it is only obstacles (such as rough wall or ground, and imperfect elasticity) which prevent exact equality. Reflection of light, showing the law to be exact. Refraction of light. Limits to both by polarisation, as before explained (art. 79, v. p. 77). Magnetic attraction, formation of needles into magnets, polarity of needles, the north of the magnet compared with position of polar star. Determination of south from the position of sun obtained by observing shadows of a pole in the play-ground, comparison of this with magnetic south, and comparisons of time of sun's southing with the clock noon. Electric attraction of rubbed sealing wax and rubbed glass for chips or pieces of cork, especially suspended by silk strings, and the subsequent repulsion. Experiments on sound, law of reflection of sound as in light, echoes, whispering tubes, scratch at end of long pole heard at other, rates of light and sound. This last should be reduced to measurement. Children at different distances clap hands when another raises his hand, the sounds are heard in succession. Measure distance (about 1120 feet, at a temperature of 60° Fahrenheit) at which sound is heard one second after the hands were seen to be clapped. Apply to thunder and lightning, to determine the distance of the storm.

It seems scarcely necessary to repeat that no one can have a proper notion of the meaning and power of induction who has not gone through at least some course of some particular science, treated in such a manner as to show the elementary observations, the primary inductions, the experiments for rendering those inductions more precise, their statement as definite assertions, the deductions drawn from these assertions, the verifications of the conclusions thereby obtained, and the precautions taken throughout to avoid error either from inaccurate observation, or imperfect ratiocination. Success in teaching depends on the teacher's own knowledge and power of communication. Book learning can at most suggest. He must have handled the tools himself. He must know what induction is, practically as well as theoretically. A few such teachers are even now to be found. May they soon be rife.

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